Finance without probability: some recent results

The standard approach to finance starts from postulating a statistical model for the prices of securities (such as the Black-Scholes model). Since such models are often difficult to justify, it is interesting to explore what can be done without making any stochastic assumptions. To my knowledge, the first results of this kind were obtained (in the case of discrete time) by Thomas Cover in 1991 and David Hobson in 1998. In this talk I will review several recent results in this direction obtained by Wouter Koolen, Steven de Rooij, Glenn Shafer, Philip Dawid, myself, and others. I will consider the simplest case of one security, without making any stochastic assumptions about its price path but making some analytic assumptions: in the first part of the talk, I will assume that the price path is non-negative and right-continuous; in the second part (if I reach it), I will assume that the price path is continuous. Two related kinds of results will be discussed: there exists a prudent (i.e., never going into debt) investment strategy that makes the investor very (or infinitely) rich if the price path is anomalous in some respect (e.g., is more volatile than Brownian motion); there is a prudent investment strategy that is competitive with a wide class of prudent investment strategies (e.g., with any prudent investment strategy that is not too complicated). The second kind of results can often be expressed in terms of option pricing.