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Finance without probability: some recent results

The standard approach to finance starts from postulating a statistical
model for the prices of securities (such as the Black-Scholes model).
Since such models are often difficult to justify, it is interesting to
explore what can be done without making any stochastic assumptions. To my
knowledge, the first results of this kind were obtained (in the case of
discrete time) by Thomas Cover in 1991 and David Hobson in 1998. In this
talk I will review several recent results in this direction obtained by
Wouter Koolen, Steven de Rooij, Glenn Shafer, Philip Dawid, myself, and
others. I will consider the simplest case of one security, without making
any stochastic assumptions about its price path but making some analytic
assumptions: in the first part of the talk, I will assume that the price
path is non-negative and right-continuous; in the second part (if I reach
it), I will assume that the price path is continuous. Two related kinds of
results will be discussed: there exists a prudent (i.e., never going into
debt) investment strategy that makes the investor very (or infinitely)
rich if the price path is anomalous in some respect (e.g., is more
volatile than Brownian motion); there is a prudent investment strategy
that is competitive with a wide class of prudent investment strategies
(e.g., with any prudent investment strategy that is not too complicated).
The second kind of results can often be expressed in terms of option
pricing.