Analyst Reputation, Communication and Information Acquisition

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Abstract

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I examine how reputational concerns affect analysts’ incentives to invest in information acquisition and their subsequent strategic communication with investors (in form of repeated cheap talk). Both biased and unbiased analysts aim to build reputation because they want investors to seriously consider their reports in the future. If analysts’ information-gathering costs are moderate, then only unbiased analysts will acquire information. As a result, investors will update favorably their beliefs about the analysts’ type when the report is consistent with the realized state. Thus, biased analysts will trade off their short-term incentives to misreport against long-term incentives to truthfully report so as to build reputation. As the analysts’ future concerns become more important, the average optimism in their reports decreases. Through truthful communication, unbiased analysts reveal their type at least partially. If information-gathering costs are small and analysts’ future concerns are important, communication during the reputation formation stage becomes uninformative. This suggests that a decrease in analysts’ information-gathering costs (e.g., due to more transparent disclosure standards) may locally reduce social welfare.
1 Introduction

Financial analysts add value in the capital market by providing information to investors. One of the stylized facts emerging from the extant literature is that analysts’ reports are on average optimistic. Many studies have proposed an incentive-based explanation for analyst optimism. Conventional wisdom and recent academic papers (e.g., Fang and Yasuda (2005), Jackson (2005)) suggest that analysts’ reputational (future) concerns may be an effective mechanism to curb opportunism. However, Morris (2001) derives the counterintuitive finding that reputational concerns may reduce the informativeness of communication in that no information is conveyed in equilibrium if analysts are sufficiently concerned about their future. The key assumption behind Morris’ surprising result is that analysts’ information sets are exogenously given and identical. In practice, analysts actively engage in information acquisition through various channels such as developing industry knowledge and analyzing financial reports. In this paper, I examine how analysts’ reputational concerns affect their incentives to acquire information and their subsequent strategic communication with investors. I find that when analysts’ information-gathering costs are moderate, only unbiased analysts acquire information. As a result, the undesirable consequences of future concerns, which Morris labels the “political correctness effect”, completely disappear.

To model the reputation formation process, I consider a repeated cheap talk game with two communication periods preceded by an information acquisition stage. In each period, the investor makes an investment decision based on information strategically communicated by the analyst. The investor is uncertain about the analyst’s type. An unbiased analyst wants the investor to make the correct investment decision. A biased analyst, in contrast, always prefers a higher investment level due to, say, underwriting considerations, trading commission incentives, or pressure from clients or covered firms.¹ Each analyst is

¹For underwriting considerations, see Dugar and Nathan (1995), Lin and McNichols (1998), Michaely and Womack (1999), Dechow, Hutton and Sloan (2000), and Hong and Kubik (2003). For trading commission incentives, see Irvine (2004), Jackson (2005), Cowen, Groysberg and Healy (2006). Jackson (2005) empirically documents that optimistic analysts generate higher trading volume. Cowen, Groysberg and Healy (2006) conclude that “analyst optimism is at least partially driven by trading incentives”. Although clients are not obligated to deal through the broker whose analyst provided the research triggering the trading decision, they often do allocate more trade through this broker in order to maintain a good relationship with the analyst. Due to the institutional restrictions and costs associated with short-selling (D’Avolio (2002)), positive reports are more effective in generating trading than negative reports. Mola and Guidolin (2009) look at pressure from clients and empirically document that sell-side analysts are likely to assign frequent and favorable ratings to a stock after the analysts’ affiliated mutual funds in-
endowed with some noisy private information about the true state of the world. At the outset, the analyst may engage in (unobservable) costly information acquisition to increase the precision of her signal for both periods. At the end of the first period, the investor updates his belief about the analyst’s type based on the analyst’s first period report and the realized state. This updated belief about the analyst’s type is labeled “analyst reputation”. The second period then unfolds similarly as the first period with a new state of the world.

To demonstrate the main result, it is useful to first examine the communication game in the second period. Since this is the last period, analysts do not care about maintaining their reputation. Consequently, the unbiased analyst will report truthfully and the biased analyst will issue a high report independent of her signal. Hence if the investor receives a low report, he learns with certainty that the analyst is unbiased. If the investor receives a high report, analyst reputation (formed in the first period) matters in that the greater is the assessed likelihood that the analyst is unbiased, the more seriously the investor will take the analyst’s report and invest accordingly. As a result, both analysts benefit from a high reputation. In addition, since the biased analyst is more likely to exploit her reputation, she benefits more from a high reputation than the unbiased analyst.

Now consider strategic communication in the first period. If both types of analysts have the same precision, Morris’ (2001) result carries over in that first period communication becomes uninformative if analysts’ future concerns are sufficiently important. Since both types of analysts benefit from a high reputation, the biased analyst wants to pool with the unbiased analyst. If both have the same precision, then the biased analyst has the ability to achieve complete pooling; when future concerns are sufficiently important, it is in fact optimal for the biased analyst to do so. However, complete pooling in terms of analysts’ types makes communication uninformative. The same argument applies, a fortiori, if the biased analyst has greater precision.

If, on the other hand, on the equilibrium path only the unbiased analyst acquires information and hence is better informed than the biased analyst, then informative (first period) communication may resurface. Now the biased analyst can no longer mimic her unbiased peer. Hence, the analysts differ effectively both in their precision and in their vest in that stock. For pressure from covered firms, see Francis and Philbrick (1993), Das, Levine and Sivaramakrishnan (1998), Lim (2001), Lambert and Sapsford (2001), and Solomon and Frank (2003).

In equilibrium, communication in the first period is either informative about both the analyst’s type and the underlying state, or informative about neither.
bias levels. Accordingly, there are two ways to build reputation in the first period: (1) by issuing a report as accurately as possible; and (2) by issuing a low report. When the improved precision is perfect, as I assume in this paper, the first mechanism dominates so that both types of analysts have reputational incentives to report truthfully.\(^3\) That is, the detrimental role of reputational concerns, documented in Morris (2001), disappears; instead, reputational concerns serve as an effective disciplining device to curb opportunistic analyst behavior. Now, the question is whether the unbiased analyst will indeed acquire information and become better informed than the biased analyst, in equilibrium.

To understand the analysts’ incentives to acquire information, note that, loosely speaking, they benefit from better information through two channels. First, it increases the analysts’ ability to build reputation. Recall that the biased analyst benefits from a high reputation even more than the unbiased analyst. Second, better information enables the analysts to guide investors towards more profitable decisions, holding reputation constant. Since the unbiased analyst internalizes the investors’ preferences, this increases the unbiased analyst’s payoff. In contrast, precision \textit{per se} does not matter to the biased analyst because her payoff is independent of the state. Combining these two arguments, it is not clear, a priori, which type of analyst benefits more from greater precision. My main result shows that overall the unbiased analyst has a stronger incentive to acquire information. Hence, if information-gathering costs are moderate, only the unbiased analyst will acquire information and informative communication will take place.

If analysts’ information-gathering costs are sufficiently small and future concerns are important, again only the unbiased analyst will acquire information, but now first period communication becomes uninformative. The reason is as follows. For sufficiently small information-gathering costs, the unbiased analyst is always better off acquiring information (because she wants the investor to make profitable decisions). At the same time, if analysts care a lot about the future, first period communication has to be uninformative because otherwise the biased analyst would also choose to acquire information for reputation building purposes. However, with both types of analysts having equal precision, as argued above, first period communication has to be uninformative. This result leads to the somewhat counterintuitive finding that a decrease in analysts’ information-gathering

\(^3\)The assumption that the improved precision is perfect helps streamline the exposition. Even if the improved precision were imperfect, both types of analysts would have reputational incentives to report truthfully when future concerns are sufficiently important.
costs may locally reduce social welfare because of its detrimental effect on information transmission.

Aside from reaffirming the beneficial role of reputational concerns in a repeated reporting setting, this paper generates several empirical predictions:

- On average, analysts’ reports are informative but optimistic. This finding is consistent with many empirical studies (e.g., Dimson and Marsh (1984), Womack (1996), O’Brien (1988), Lys and Sohn (1990), Brown (1993), Dugar and Nathan (1995)).

- As analysts’ future concerns gain in importance, the average optimism in analysts’ reports decreases. NYSE Rule 472(j)(2) encourages analysts to make their track record more transparent, which in turn helps investors (especially small investors) form an opinion about analysts’ type. The model suggests that the increased future concerns might be a confounding factor for the documented decrease in analyst optimism subsequent to the 2002-2003 regulatory changes.\(^4\)

- In general, I find that unbiased analysts tend to have more precise information and are more likely to report truthfully. Hence, my model predicts the endogenous association that analysts with higher forecast accuracy are less optimistic. This prediction is confirmed by Conroy and Harris (1995). However, the contrary view represented by Lim (2001) and Chen and Matsumoto (2006) suggests that more optimistic analysts may have better access to management inside information, and hence will have higher forecast accuracy. Regulation FD reduces managers’ ability to selectively provide or withhold information to different analyst groups, alleviating the impact of management access on analysts’ information sets. Hence, I expect that my model predictions speak more directly to the post-Reg FD regime.

- Higher reputation analysts have greater impact on investors’ decisions. This prediction is confirmed by Stickel (1992), Park and Stice (2000), Jackson (2005), and Chang, Daouk and Wang (2008).

\(^4\)Before the 2002-2003 regulatory changes, the conflict of interest between investment banking and research departments of U.S. brokerage firms was of great concern. The purpose of these regulatory changes (the Global Settlement, the contemporaneous NASD Rule 2711 and amended NYSE Rule 472) was to curb this conflict of interest by substantially limiting the relations between research and investment banking departments of U.S. brokerage firms.
• Higher reputation analysts have better future performance. This, too, is consistent with Stickel (1992) and Desai, Liang and Singh (2000), among others.

• One way to proxy for analysts’ bias, at least before the Global Settlement, is to see whether analysts’ employers have underwriting relationships with the covered firms. My model predicts that affiliated analysts issue more optimistic reports than unaffiliated analysts. Not surprisingly, this finding is confirmed by most empirical studies (e.g., Dugar and Nathan (1995), Lin and McNichols (1998), Michaely and Womack (1999), Dechow, Hutton and Sloan (2000)).

• More surprisingly, the model suggests that for small information-gathering costs (e.g., due to high disclosure standards) and important future concerns, analysts’ reports will be less informative.

Related Literature. This paper belongs to the cheap talk literature initiated by Crawford and Sobel (1982). Focusing on the reputation dynamics, Sobel (1985), Benabou and Laroque (1992), Kim (1996), Stocken (2000), Morris (2001) and Wang (2009) study repeated cheap talk games and examine how future concerns affect communication. In particular, Sobel (1985) and Benabou and Laroque (1992) show that sometimes a bad advisor (with the opposite interest of the decision maker) will tell the truth in order to build reputation. However, as previously mentioned, in a setting similar to mine, but with exogenous and identical precision for both unbiased and biased advisors,5 Morris (2001) shows that the unbiased advisor may have an incentive to lie in order to build reputation. As a result, no information is conveyed in equilibrium if the advisor is sufficiently concerned about her reputation.

The effect of reputation on the analysts’ communication behavior in a static model is the focus of Trueman (1994) and Jackson (2005). Trueman (1994) finds that in order to enhance investors’ assessment of their forecasting abilities, analysts tend to release forecasts closer to prior expectations than is warranted given their private information, and analysts with less ability are more likely to herd. Jackson (2005) examines how

5There are two other (lesser) differences in the modeling: (1) the biased analyst’s payoff takes the form of a quadratic loss function in my paper, while Morris (2001) deals with a linear payoff function for the biased analyst; (2) in my model, both analysts have the same future concerns (1/x), while Morris (2001) allows different analysts to have different future concerns. Because I endogenize the analyst’s precision choice, the payoff structure and the future concerns of the biased analyst should be comparable with those of the unbiased analyst.
analysts trade off short-term incentives to generate more trade against long-term gains from building reputation. In both papers, analysts’ reputation (type) is with regard to their precision, which is exogenously given. Whereas, in this paper, I allow analysts to affect the precision of their information. In addition, the reputation benefit function is exogenous in the static models.

Prior research has studied analysts’ communication and information gathering behavior without future concerns. Morgan and Stocken (2003) study information transmission between analysts and investors when investors are uncertain about analysts’ incentives and analysts’ information sets are exogenous.\(^6\) Hayes (1998) examines how incentives to generate commissions affect analysts’ information gathering decisions, but she assumes that analysts report truthfully. Fischer and Stocken (2010) endogenize both analysts’ information gathering and their reporting behavior. They investigate how public information affects analysts’ information gathering decisions and the communication with investors.

The remainder of the paper is organized as follows: Section 2 lays out the model. Section 3 studies the communication game in each period for exogenous and commonly known analysts’ precision. Section 4 fully characterizes the equilibrium of the model where analysts’ precision choices are endogenous and unobservable. Section 5 discusses the welfare consequences of changes in information-gathering costs. Section 6 presents the empirical implications and Section 7 concludes. All proofs are contained in the Appendix.

2 Model Setup

In this section, I describe the basic setup of the model, which follows Morris (2001). I consider an investor (“he”) who is uninformed about the state of the world and makes decisions based on the advice provided by an analyst (“she”). With probability \(\lambda\), the analyst is unbiased (\(U\)); that is, she wants the investor to make correct investment decision in each period. With probability \(1 - \lambda\), the analyst is biased (\(B\)) and always wants the investor to make “buy” decisions (independent of the state of the world). The investor is uncertain about the analyst’s type \(J \in \{U, B\}\) and only knows the prior probability of the analyst being unbiased (\(\lambda\)).

The game has one information acquisition stage and two communication periods. At

\(^6\)Beyer and Guttman (2007) study the interaction between the analyst and the investor in a signalling model. In their model, reputational concerns constitute part of analysts’ misreporting costs.
stage 0, the analyst may choose to exert unobservable effort $c$ to increase the precision of her signal for the following two periods. In period 1, the state of the world $w_1$ can take the value of 0 or 1; each state occurs with equal probability. The analyst observes an informative signal $s_1 \in \{0, 1\}$ about the state of the world, and the default precision is $\tilde{\gamma} : Pr(s_1 = w_1 | w_1) = \tilde{\gamma} \in (1/2, 1)$. If the analyst acquires information at stage 0, she will become perfectly informed in the subsequent two periods. After observing the signal, the analyst issues a report $m_1 \in \{0, 1\}$. The investor then makes an investment decision $a_1 \in R$ according to his inference about the state based on the analyst’s report $m_1$. After the action $a_1$ is taken, the state of the world $w_1$ is publicly observed. Then the investor updates his belief about the analyst’s type based on the realized state $w_1$ and the received report $m_1$. As a result, the analyst now has reputation $\lambda_2 = \Lambda(m_1, w_1)$ (to be specified below) entering period 2. Period 2 then unfolds similarly to period 1, with a new and independent state $w_2$ (again equally likely to be 0 or 1), a new signal $s_2$, a new report $m_2$ sent by the analyst, and a new action $a_2$ taken by the investor.

The sequence of events is as follows:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>information acquisition stage</td>
<td>1st communication period</td>
<td>2nd communication period</td>
</tr>
<tr>
<td>Analyst chooses $\gamma^J$</td>
<td>Analyst observes $s_1$</td>
<td>Analyst observes $s_2$</td>
</tr>
<tr>
<td>Analyst reports $m_1$</td>
<td>Investor chooses $a_1$</td>
<td>Analyst reports $m_2$</td>
</tr>
<tr>
<td>State $w_1$ is observed</td>
<td>Investor updates belief</td>
<td>Investor chooses $a_2$</td>
</tr>
</tbody>
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Figure 1: Timeline

In each period, the investor aims to adjust his investment decision $a_t$ to the state of the world $w_t$. His utility in each period, $t$, is given by a quadratic loss function

$$-(a_t - w_t)^2.$$ 

The unbiased analyst has identical preferences over $a_t$ as the investor. The utility of the unbiased analyst is given by

$$-x(a_1 - w_1)^2 - (a_2 - w_2)^2 - C(\gamma^U).$$
The biased analyst, in contrast, always wants the high action to be chosen, independent of the state. Her utility is given by

$$-x(a_1 - 1)^2 - (a_2 - 1)^2 - C(\gamma B),$$

where \(x > 0\) captures the weight both types of analysts put on period 1 compared with period 2 payoffs. I refer to \(1/x\) as the exogenous analyst’s future (reputational) concerns.\(^7\) \(C(\cdot)\) represents each analyst’s disutility of acquiring certain precision, with \(C(\gamma) = 0\) and \(C(1) = c\). Note that the cost of acquiring information is assumed to be independent of the analyst’s type.

An equilibrium in this game is characterized by the analyst’s information acquisition strategy at stage 0, the analyst’s communication strategy in each period, the decision rule for the investor in each period, and the belief function of the investor. The type \(J\) analyst’s information acquisition strategy specifies the precision she will choose at stage 0; I denote it by \(\gamma^J \in \{\bar{\gamma}, 1\}\). The type \(J\) analyst’s communication strategy in period \(t\) is a function \(\sigma^J_t : \{0, 1\} \times \{\bar{\gamma}, 1\}^3 \to [0, 1]\), where \(\sigma^J_t(s_t, \gamma^J | \tilde{\gamma})\) is the probability of the type \(J\) analyst reporting 1 in period \(t\) when her signal is \(s_t\) and her precision is \(\gamma^J\) while the investor’s conjecture of the analyst’s precision is \(\tilde{\gamma} \equiv (\tilde{\gamma}^U, \tilde{\gamma}^B)\).\(^8\) The investor’s decision rule in period \(t\) is a function \(a_t : \{0, 1\} \times \{0, 1\} \times \{\bar{\gamma}, 1\}^2 \to R\), where \(a_t(m_t, \lambda_t, \tilde{\gamma})\) is the investor’s action in period \(t\) when he receives message \(m_t\), his belief of the analyst being unbiased is \(\lambda_t\) and his conjecture of the analyst’s precision is \(\tilde{\gamma}\). As is implied by the notation, I only consider pure strategy for the analyst’s information acquisition decision; however I do allow the analyst to play mixed communication strategies.

Let \(\phi^J_t(m_t | w_t)\) denote the investor’s conjecture about the probability of the type \(J\) analyst sending message \(m_t\) given state \(w_t\) in period \(t\):

$$\phi^J_t(1 | w_t) = \tilde{\gamma}^J \tilde{\sigma}^J_t(w_t, \tilde{\gamma}^J | \tilde{\gamma}) + (1 - \tilde{\gamma}^J) \tilde{\sigma}^J_t(1 - w_t, \tilde{\gamma}^J | \tilde{\gamma}),$$

and \(\phi^J_t(0 | w_t) = 1 - \phi^J_t(1 | w_t)\). The belief function \(\Gamma_t(m_t, \lambda_t, \tilde{\gamma})\) states the investor’s inference of the actual state being 1 in period \(t\). By Bayes rule, it is given by

$$\Gamma_t(m_t, \lambda_t, \tilde{\gamma}) = \frac{\lambda_t \phi^U_t(m_t | 1) + (1 - \lambda_t) \phi^B_t(m_t | 1)}{\lambda_t \phi^U_t(m_t | 1) + (1 - \lambda_t) \phi^B_t(m_t | 1) + \lambda_t \phi^U_t(m_t | 0) + (1 - \lambda_t) \phi^B_t(m_t | 0)}.$$  \hspace{1cm} (1)

\(^7\)The two periods are not necessarily of equal length, so \(x < 1\) is possible.

\(^8\)Strictly speaking, the analyst’s communication strategy should depend on her conjecture about the investor’s action. However, the analyst will infer that the investor’s action depends on his conjecture about the analyst’s precision, \(\tilde{\gamma} \equiv (\tilde{\gamma}^U, \tilde{\gamma}^B)\). Hence, I write out \(\tilde{\gamma}\) instead of \(\tilde{\alpha}_t(\cdot)\).
Γ_t(m_t, λ_t, ˜γ) is well defined when the denominator is nonzero. I adopt the convention that Γ_t(m_t, λ_t, ˜γ) = 1/2 if the denominator is zero. That is, when the posterior belief of the state is undefined according to Bayes rule, the investor keeps his prior belief about the state. At the end of period 1, the investor updates his belief about the analyst’s type. In particular, λ_1 = λ is the prior reputation, and λ_2 = Λ(m_1, w_1| ˜γ) denotes the posterior reputation, defined as the investor’s belief of the analyst being unbiased if report m_1 is received and state w_1 is realized:

\[ \Lambda(m_1, w_1| ˜γ) = \frac{λφ^U_1(m_1|w_1)}{λφ^U_1(m_1|w_1) + (1 - λ)φ^B_1(m_1|w_1)} . \] (2)

Again, I adopt the convention that Λ(m_1, w_1| ˜γ) = λ, the prior, if the denominator is zero.

At this point, I am in a position to define the equilibrium of the game.\(^9\)

**Definition 1** A Perfect Bayesian Nash equilibrium of the game is a strategy-belief profile \((γ^U, γ^B, σ^U_t(·), σ^B_t(·), a_t(·), Γ_t(·), Λ(·))\) satisfying the following properties:

1. The communication strategy of the type \(J\) analyst in period \(t\), \(σ^J_t(s_t, γ^J_t)\), maximizes her utility in period \(t\) given \(a_t(m_t, λ_t)\).

2. The investor’s action in period \(t\), \(a_t(m_t, λ_t)\), is optimal given the state inference function \(Γ_t(m_t, λ_t)\).

3. The information acquisition strategy of the type \(J\) analyst, \(γ^J_t\), maximizes her utility at the information acquisition stage.

4. The state and type inference functions, \(Γ_1(m_1, λ)\), \(Γ_2(m_2, Λ)\) and \(Λ(m_1, w_1)\), are derived from the analyst’s equilibrium strategy according to inference rules (1) and (2).

To facilitate the following arguments, I formally define “informative” communication in period \(t\):

**Definition 2** (1) Communication in period 1 takes the form of “babbling” if \(Γ_1(0, λ) = Γ_1(1, λ) = \frac{1}{2}\) and \(Λ(1, 1) = Λ(0, 1) = Λ(1, 0) = Λ(0, 0) = λ\).

(2) Communication in period 2 is babbling if \(Γ_2(0, λ_2) = Γ_2(1, λ_2) = \frac{1}{2}\).

(3) Communication in period \(t\) is “informative” if and only if it is not babbling.

\(^9\)When I define the equilibrium here, I suppress the functional dependence of the players’ strategies and belief functions on \( ˜γ = ( ˜γ^U, ˜γ^B) \). Such conjecture will be borne out in equilibrium. The formal definition of the equilibrium is relegated to the Appendix.
Communication in the first period may be informative in terms of either the analyst’s type or the underlying state, whereas the only relevant dimension of informativeness in the second period is with regard to the underlying state. In the following, I refer to an equilibrium in which communication in each period is babbling as a babbling equilibrium, and an equilibrium in which communication in either period is informative as an informative equilibrium.

Extending standard arguments from the cheap talk literature in which babbling equilibria always exist, in my model there always exists an equilibrium in which neither analyst acquires information and communication in each period is babbling. Suppose that in each period the analyst issues report randomly, independent of her type and signal. Then the investor will rationally make his investment decision solely based on his prior knowledge of the state. Given such response of the investor, the analyst has incentive neither to deviate from her uninformative report, nor to become better informed. Therefore, a babbling equilibrium always exists and neither analyst will acquire information. The interesting question is whether and when there exist informative equilibria and which, if any, type of analyst chooses to acquire information. In the following analysis, without loss of generality, I assume \( a_t(1, \lambda_t, \tilde{\gamma}) \geq a_t(0, \lambda_t, \tilde{\gamma}) \).

3 The Repeated Communication Game — Exogenous and Commonly Known Precision

For now, to illustrate the key features of the communication game, I take the analyst’s precision \( \gamma^J \) as exogenously given and commonly known; I will relax this assumption in Section 4. The communication game can be solved by backward induction.

3.1 The Second Period Communication Game

At the end of period 1, the investor updates his belief about the analyst’s type according to (2) and the analyst now has a commonly known reputation, \( \lambda_2 \), entering period 2. Since period 2 is the last period, each type of analyst has no incentive to protect her reputation and simply seeks to maximize her utility in that period.

In line with the cheap talk literature, I assume that informative communication, if it can be supported in equilibrium, is played in each period. The following arguments show that pure strategy informative communication always obtains in the second period.
Suppose this is the case, then \( a_2(1, \lambda_2, \gamma) > a_2(0, \lambda_2, \gamma) \). Therefore the biased analyst has a strict incentive to report 1, and the unbiased analyst must have a strict incentive to report her signal truthfully. If the investor receives message 0, he will be sure that the analyst is unbiased and truthfully reporting her signal. Given the unbiased analyst’s precision, \( \gamma^U \), the investor will assign probability \( 1 - \gamma^U \) to state 1 and choose action \( a_2(0, \lambda_2, \gamma) = 1 - \gamma^U < 1/2 \). If the investor receives message 1, he will be uncertain about the analyst’s type and choose his action based on the updated belief:

\[
a_2(1, \lambda_2, \gamma) = \frac{\frac{1}{2}[\lambda_2 \gamma^U + (1 - \lambda_2)]}{\frac{1}{2}[\lambda_2 \gamma^U + (1 - \lambda_2)]} = \frac{1 - \lambda_2 + \lambda_2 \gamma^U}{2 - \lambda_2}.
\]

Clearly, \( a_2(1, \lambda_2, \gamma) \in [1/2, \gamma^U] > a_2(0, \lambda_2, \gamma) \). Therefore, the biased analyst will indeed always report 1. It is also shown that the unbiased analyst will indeed truthfully report her signal. Hence pure strategy informative communication does obtain in the second period.

In addition, all else equal, the action induced by report 1, \( a_2(1, \lambda_2, \gamma) \), is increasing in analyst reputation \( \lambda_2 \). The higher the probability an analyst is believed to be unbiased, the more credible her report is perceived to be, and hence the investor will choose a higher action accordingly.

Given analyst reputation \( \lambda_2 \), write \( V^J(\lambda_2, \gamma) \) for the type \( J \) analyst’s second period expected utility when the analyst’s precision is \( \gamma \equiv (\gamma^U, \gamma^B) \). The unbiased analyst’s

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10In Section 3, since the analyst’s precision \( \gamma^J \) is exogenous and commonly known, the players’ strategies and belief functions depend no longer on their conjectures about the analyst’s precision, instead they depend on the commonly known \( \gamma = (\gamma^U, \gamma^B) \).

11The argument is as follows: given that the biased analyst reports 1 all the time, for \( a_2(1, \lambda_2, \gamma) > a_2(0, \lambda_2, \gamma) \) to hold, the unbiased analyst must report 1 more often when she observes signal 1 than when she observes signal 0. Since I focus here on pure strategies, this means the unbiased analyst must report her signal truthfully.

12This confirms Morgan and Stocken’s (2003) finding that the investor’s uncertainty about the analyst’s incentive makes it impossible for the unbiased analyst to credibly reveal good news.

13If the unbiased analyst observes signal 0, she will compare her payoff conditional on sending message 0, \( U^B_2(m_2 = 0, s_2 = 0, \lambda_2, \gamma^U) = -\gamma^U(a_2(0, \lambda_2, \gamma) - 0)^2 - (1 - \gamma^U)(a_2(0, \lambda_2, \gamma) - 1)^2 \), with her payoff conditional on sending message 1, \( U^B_2(m_2 = 1, s_2 = 0, \lambda_2, \gamma^U) = -\gamma^U(a_1(1, \lambda_2, \gamma) - 0)^2 - (1 - \gamma^U)(a_1(1, \lambda_2, \gamma) - 1)^2 \). It is straightforward to show that \( U^B_2(m_2 = 0, s_2 = 0, \lambda_2, \gamma^U) - U^B_2(m_2 = 1, s_2 = 0, \lambda_2, \gamma^U) = (a_2(1, \lambda_2, \gamma) - a_2(0, \lambda_2, \gamma))(a_2(1, \lambda_2, \gamma) - a_2(0, \lambda_2, \gamma) - 2(1 - \gamma^U)) > 0 \). Hence the unbiased analyst will indeed report 0 when she observes signal 0. If, on the other hand, the unbiased analyst observes signal 1, she will compare her payoff conditional on sending message 0, \( U^B_2(m_2 = 0, s_2 = 1, \lambda_2, \gamma^U) = -\gamma^U(a_2(0, \lambda_2, \gamma) - 1)^2 - (1 - \gamma^U)(a_2(0, \lambda_2, \gamma) - 0)^2 \), with her payoff conditional on sending message 1, \( U^B_2(m_2 = 1, s_2 = 1, \lambda_2, \gamma^U) = -\gamma^U(a_1(1, \lambda_2, \gamma) - 1)^2 - (1 - \gamma^U)(a_1(1, \lambda_2, \gamma) - 0)^2 \). Then \( U^B_2(m_2 = 0, s_2 = 1, \lambda_2, \gamma^U) - U^B_2(m_2 = 1, s_2 = 1, \lambda_2, \gamma^U) = (a_2(1, \lambda_2, \gamma) - a_2(0, \lambda_2, \gamma))(a_2(1, \lambda_2, \gamma) - a_2(0, \lambda_2, \gamma) - 2\gamma^U) < 0 \). The inequality stems from the fact that \( \gamma^U > a_2(1, \lambda_2, \gamma) > a_2(0, \lambda_2, \gamma) \).

14In fact it can be shown that this pure strategy informative equilibrium is the unique informative equilibrium in the second period.
second period expected utility (anticipating she will report truthfully) is therefore:

\[
V^U(\lambda_2, \gamma) = -\frac{1}{2} \gamma^U[a_2(1, \lambda_2, \gamma) - 1]^2 - \frac{1}{2} (1 - \gamma^U)[a_2(0, \lambda_2, \gamma) - 1]^2 \\
- \frac{1}{2} (1 - \gamma^U)[a_2(1, \lambda_2, \gamma) - 0]^2 - \frac{1}{2} \gamma^U[a_2(0, \lambda_2, \gamma) - 0]^2 \\
= -\frac{(\lambda_2 - 1)^2 + 2\gamma^U(\gamma^U - 1)(\lambda_2^2 - 2)}{2(\lambda_2 - 2)^2}. \tag{3}
\]

The biased analyst’s second period expected utility equals:

\[
V^B(\lambda_2, \gamma) = -[a_2(1, \lambda_2, \gamma) - 1]^2 \\
= -\frac{2(\gamma^U - 1)(1 - \lambda_2\gamma^U)}{(\lambda_2 - 2)^2}. \tag{4}
\]

It is straightforward to show that

\[
\frac{\partial V^U(\lambda_2, \gamma)}{\partial \lambda_2} = \frac{(1 - \lambda_2)(2\gamma^U - 1)^2}{(2 - \lambda_2)^3} \geq 0,
\]

\[
\frac{\partial V^B(\lambda_2, \gamma)}{\partial \lambda_2} = \frac{2(2\gamma^U - 1)(1 - \lambda_2\gamma^U)}{(2 - \lambda_2)^3} \geq 0.
\]

Also

\[
\frac{\partial V^U(\lambda_2, \gamma)}{\partial \lambda_2} - \frac{\partial V^B(\lambda_2, \gamma)}{\partial \lambda_2} = -\frac{(2\gamma^U - 1)(3 - 2\gamma^U - \lambda_2)}{(2 - \lambda_2)^3} < 0.
\]

Both types of analysts benefit from a high reputation, with the biased analyst benefiting even more. To generate intuition for this result, notice that the investor’s action upon receiving message 1 is increasing in analyst reputation \(\lambda_2\), while the action induced by message 0 is independent of \(\lambda_2\). The biased analyst always reports 1 and hence her reputation pays off in all scenarios. In contrast, with (ex-ante) probability 1/2, the unbiased analyst reports 0, in which case her payoff is independent of her reputation, \(\lambda_2\). Only with the remaining probability, the unbiased analyst reports 1 and benefits from her reputation. Therefore, the biased analyst is more likely to exploit her reputation and hence benefits more from a high reputation than her unbiased peer.

To conclude, in the second period, the unbiased analyst reports truthfully and the biased analyst always reports 1. Both types of analysts benefit from a high reputation with the biased analyst benefiting from it even more than the unbiased one.

### 3.2 The First Period Communication Game

In the first communication period, the analyst takes into consideration the reputational consequences of the second period when choosing her communication strategy. Specifically,
the unbiased analyst’s objective in the first communication period now includes both her first period payoff and her second period expected utility, and is given by

\[-x(a_1 - w_1)^2 + V^U(\Lambda(m_1, w_1|\gamma), \gamma).\]

Analogously the biased analyst’s objective in the first communication period is given by

\[-x(a_1 - 1)^2 + V^B(\Lambda(m_1, w_1|\gamma), \gamma).\]

To develop the argument, I adapt Morris’ (2001) notation to my setting. Write $\Pi^J_C(s_1|\gamma)$ for the net current expected gain to the type $J$ analyst of reporting 1 rather than 0, when she observes signal $s_1$ and the analyst’s precision $\gamma = (\gamma^U, \gamma^B)$:

$$
\Pi^J_C(s_1|\gamma) = -x\{\gamma^U(a_1(1, \gamma) - s_1)^2 + (1 - \gamma^U)(a_1(1, \gamma) - (1 - s_1))^2\} + x\{\gamma^U(a_1(0, \gamma) - s_1)^2 + (1 - \gamma^U)(a_1(0, \gamma) - (1 - s_1))^2\},
$$

$$
\Pi^B_C(1|\gamma) = \Pi^B_C(0|\gamma) = -x(a_1(1, \gamma) - 1)^2 + x(a_1(0, \gamma) - 1)^2.
$$

(5)

Write $\Pi^J_R(s_1|\gamma)$ for the net expected reputational gain to the type $J$ analyst of reporting 0 rather than 1, when she observes signal $s_1$ and the analyst’s precision is $\gamma$:

$$
\Pi^J_R(s_1|\gamma) = \gamma^J[V^J(\Lambda(0, s_1|\gamma), \gamma) - V^J(\Lambda(1, s_1|\gamma), \gamma)] + (1 - \gamma^J)[V^J(\Lambda(0, 1 - s_1|\gamma), \gamma) - V^J(\Lambda(1, 1 - s_1|\gamma), \gamma)].
$$

(6)

The type $J$ analyst has a strict incentive to report 1 when she observes $s_1$ if and only if $\Pi^J_C(s_1|\gamma) > \Pi^J_R(s_1|\gamma)$. I refer to $\Pi^J_C(\cdot)$ and $\Pi^J_R(\cdot)$ as the type $J$ analyst’s current reporting incentive and reputational reporting incentive, respectively.

According to definition 2, first period communication could be informative in terms of either the analyst’s type or the underlying state. I argue that in equilibrium, informative communication in the first period has to convey information about both dimensions. Suppose communication in the first period were uninformative about the analyst’s type, yet informative about the state. Then, there would be no reputational reporting incentive for the analyst. However, because information about the state is conveyed, $a_1(1, \gamma) > a_1(0, \gamma)$ has to hold. Then, the unbiased analyst would tell the truth, while the biased analyst would always report 1. However, these optimal reporting strategies themselves are informative about the analyst’s type, a contradiction. Therefore informative communication

15To save on notation, from now on, I suppress the functional dependence of $a_1(\cdot)$ and $\Gamma_1(\cdot)$ on $\lambda$.
in the first period must convey information about the analyst’s type. In the Appendix I show that first period communication that is informative about the analyst’s type in equilibrium also conveys information about the underlying state.

The striking finding of Morris (2001) is that when the analyst’s future concerns are sufficiently important, i.e., \( x \) becomes small, then no information can be conveyed in the first period. Similarly, in my setting, if both types of analysts have the same precision, then if the future weighs heavily, only babbling communication obtains in the first period:

**Lemma 1 (Morris 2001)**

If both types of analysts have the same precision, i.e., \( \gamma^U = \gamma^B \), then when the analyst’s future concerns are sufficiently important, i.e., \( x \to 0 \), communication in the first period is babbling.

To convey the intuition for this result, recall that both analysts benefit from a high reputation. Hence if first period communication were informative, then the biased analyst would like to pool with her unbiased peer. If both types of analysts have the same precision, then the biased analyst has the ability to pool with the unbiased analyst. Moreover, when future concerns are sufficiently important and hence current reporting incentives become immaterial, it is in fact optimal for the biased analyst to do so. However, complete pooling in terms of analysts contradicts the properties of informative communication, as argued above. Thus, when future concerns are sufficiently important and different types of analysts are equally well informed, first period communication has to take the form of babbling. In particular, if both analysts become perfectly informed (\( \gamma^U = \gamma^B = 1 \)), the threshold of future concerns can be solved for analytically as

\[
x^o = \frac{(4-3\lambda)^\lambda}{4(\lambda-2)^2},
\]

i.e., when \( x < x^o \), first period communication is babbling.\(^{16}\)

If the biased analyst has more precise information than the unbiased analyst, then the above argument applies, a fortiori. Hence, first period communication must be babbling for future concerns sufficient important. I now ask the central question for the remainder

\(^{16}\)On a technical note, this result depends crucially on the out-of-equilibrium beliefs. If, as Morris (2001) discusses, the out-of-equilibrium belief is such that the investor infers that any analyst whose message is not equal to the realized state is surely bad, then both types of analysts reporting truthfully can constitute an equilibrium. However, if the out-of-equilibrium belief is the conventional one where the investor sticks to the prior when the posterior beliefs are undefined, then both types of analysts reporting truthfully does not constitute an equilibrium. In this paper I adopt the conventional assumption on the out-of-equilibrium belief since the results it leads to are consistent with a more general model where the improved precision is sufficiently high but imperfect.
of this section: if the unbiased analyst has greater precision, can informative (first period) communication obtain in equilibrium?

**Proposition 1** If it is common knowledge that the unbiased analyst is better informed than the biased analyst, i.e., \( \gamma^U = 1 > \gamma^B = \bar{\gamma} \), then informative communication obtains in equilibrium in the first period for any level of future concerns \((1/x)\). Any such informative communication has to satisfy the following properties:

1. The investor updates favorably his belief about the analyst’s type when the analyst’s report is consistent with the realized state. More specifically,  
   \[
   \Lambda(m_1 = 1, w_1 = 1 \mid \gamma^U = 1 > \gamma^B = \bar{\gamma}) \geq \Lambda(m_1 = 0, w_1 = 1 \mid \gamma^U = 1 > \gamma^B = \bar{\gamma}),
   \]
   \[
   \Lambda(m_1 = 0, w_1 = 0 \mid \gamma^U = 1 > \gamma^B = \bar{\gamma}) \geq \Lambda(m_1 = 1, w_1 = 0 \mid \gamma^U = 1 > \gamma^B = \bar{\gamma}),
   \]
   and at least one of the inequalities is strict.\(^{17}\)

2. The unbiased analyst always reports truthfully.

3. The biased analyst reports 1 when she observes signal 1. If she observes signal 0, her probability of reporting 1 depends on the level of future concerns \((1/x)\). There exist values \(\underline{x}\) and \(\bar{x}\) such that: if future concerns are important, i.e., \(x < \underline{x}\), then the biased analyst truthfully reports 0; if future concerns are small, i.e., \(x > \bar{x}\), then the biased analyst reports 1; if \(\underline{x} \leq x \leq \bar{x}\), then the biased analyst randomizes between reporting 0 and 1.

The values of \(\underline{x}\) and \(\bar{x}\) are derived in the Appendix. If the unbiased analyst is better informed than the biased analyst, then informative communication resurfaces in the first period because the biased analyst can no longer perfectly mimic her unbiased peer.

For an unbiased analyst who has an informational advantage, there are two ways to build reputation: (1) by issuing a report as *accurately* as possible; and (2) by issuing a *low* report, since it is commonly known that the biased analyst is upwardly biased. If the realized state is low, both mechanisms suggest the investor will update favorably his belief about the analyst’s type when the report is low. If the realized state is high, the first

\(^{17}\)\(\Lambda(1,1 \mid \gamma^U > \gamma^B) \geq \Lambda(0,1 \mid \gamma^U > \gamma^B)\) holds for any \(x\) if \(\gamma^U = 1\). If \(\gamma^U < 1\), then the relation between \(\Lambda(1,1 \mid \gamma^U > \gamma^B)\) and \(\Lambda(0,1 \mid \gamma^U > \gamma^B)\) depends on \(x\). When \(x\) is sufficiently small such that both types of analysts tell the truth, then \(\Lambda(1,1 \mid \gamma^U > \gamma^B) > \Lambda(0,1 \mid \gamma^U > \gamma^B)\) holds. When \(x\) is sufficiently large such that the biased analyst always reports 1, then \(\Lambda(1,1 \mid \gamma^U > \gamma^B) < \Lambda(0,1 \mid \gamma^U > \gamma^B)\) holds instead.
mechanism suggests the investor will update his belief favorably when having received a high report, while the second mechanism suggests the reverse. When the unbiased analyst is perfectly informed, the first mechanism dominates and the investor will update his belief favorably when the analyst’s report is consistent with the state.\textsuperscript{18}

The unbiased analyst now has both current and reputational incentives to tell the truth and hence will report her signal truthfully. On the other hand, the biased analyst will report truthfully when observing signal 1. Upon observing signal 0, the biased analyst will trade off her current reporting incentive to lie against a reputational reporting incentive to tell the truth. When future concerns are sufficiently important, the reputational reporting incentive dominates and hence the biased analyst will truthfully report 0. When reputation matters less, the reverse holds.

To conclude, for exogenous and commonly known precision, if the biased analyst has equal or higher precision than the unbiased analyst and future concerns are sufficiently important, first period communication is babbling. On the other hand, if the unbiased analyst is better informed, informative communication always obtains in the first period. In the following section, I endogenize the analyst’s precision choice and assume it is unobservable to the investor.

4 Unobservable Choice of Precision

In this section, I refer back to my original model where the analyst’s precision choice is endogenous and unobservable. First, note that the communication game is not a proper subgame because the analyst’s precision choice is unobservable, i.e., the communication game starts from a non-singleton information set. However, any informative communication in the full game has to satisfy the properties characterized in Section 3 (when the analyst’s precision is exogenous and commonly known). The reason is that in equilibrium the investor’s conjecture about the analyst’s precision is consistent with the analyst’s actual precision choice. Hence, although the analyst’s chosen precision is unobservable, in equilibrium it is commonly known.

Clearly, when future concerns are sufficiently small, the first period communication game converges to the one-period version. Thus, in each period, the unbiased analyst

\textsuperscript{18}Even if the unbiased analyst’s precision is imperfect, when the precision difference ($\gamma^U - \gamma^B$) is not trivial, both types of analysts would have reputational incentives to report truthfully when future concerns are important.
reports truthfully and the biased analyst always reports 1. Hence the biased analyst has no incentive to acquire information because in each period her report is independent of her signal. In contrast, the unbiased analyst will acquire information if and only if information-gathering costs are not too large. The interesting question to consider is what kind of equilibria exist when future concerns are sufficiently important?

If information-gathering costs are very high, then neither type of analyst will acquire information. In this case, if reputation matters a lot, by Lemma 1, first period communication has to be babbling.

If information-gathering costs are moderate, additional analysis is needed to understand which analyst will acquire information. To see this, note that, loosely speaking, the analyst benefits from acquiring information through two channels. First, it increases the analyst’s ability to build reputation. Recall that while both types of analysts benefit from a high reputation, the biased analyst benefits even more so than the unbiased analyst. Second, more precise information enables the analyst to guide the investor towards more profitable decisions, holding reputation constant. Since the unbiased analyst internalizes the investor’s preferences, this increases the unbiased analyst’s payoff. In contrast, precision, per se, does not matter to the biased analyst because her payoff is independent of the state. For later reference, I label the second channel the “precision effect”. Combining these two arguments, it is not clear, a priori, which type of analyst has a stronger incentive to acquire information. The following proposition evaluates this trade-off. To that end, denote by $\Delta^J$ the type $J$ analyst’s benefit from acquiring information. I also define $\bar{\Delta} \equiv \min\{\frac{1}{2}, \frac{1}{8}\}$ and $\Delta \equiv \frac{1}{4}$. Clearly $\bar{\Delta} > \Delta$ since $\bar{\gamma} > 0.5$.

**Proposition 2** If information-gathering costs are moderate, i.e., $\Delta \leq c \leq \bar{\Delta}$, then for all levels of future concerns ($1/x$), there exists an informative equilibrium in which only the unbiased analyst acquires information and communication in each period is informative. Specifically, first period informative communication satisfies the properties characterized in Proposition 1. In the second period, the unbiased analyst reports truthfully and the biased analyst always reports 1.

This proposition states a sufficient condition for there to exist an informative equilibrium in which only the unbiased analyst acquires information and communication in each period is informative. Similar arguments as those presented in Section 3.1 and Proposition 1 show that if the investor conjectures that only the unbiased analyst acquires
information and that communication in each period is informative, then the optimal communication strategies of the analyst with conjectured precision are indeed consistent with these conjectures. Now it remains to show that for \( \Delta \leq c \leq \bar{\Delta} \), the analyst’s precision choice is consistent with the investor’s conjecture. The following arguments will show that \( \Delta^U > \bar{\Delta} > \Delta > \Delta^B \). To that end, write \( V^J(\lambda_2, \gamma^J, \tilde{\gamma}^U) \) for the type \( J \) analyst’s second period expected utility when her reputation is \( \lambda_2 \), actual precision is \( \gamma^J \) and the investor’s conjecture of the unbiased analyst’s precision is \( \tilde{\gamma}^U \).\(^\text{19}\)

By the logic presented in Section 3.1 and Proposition 1, if the investor conjectures that \( \tilde{\gamma}^U = 1 > \tilde{\gamma}^B = \bar{\gamma} \), then the unbiased analyst with conjectured precision \( \gamma^U = \tilde{\gamma}^U = 1 \) will report her signal truthfully in each period. Therefore the unbiased analyst’s total benefit from acquiring information is bounded from below by her second period precision effect:

\[
\Delta^U > V^U(\lambda_2, \gamma^U = 1, \tilde{\gamma}^U = 1) - V^U(\lambda_2, \gamma^U = \bar{\gamma}, \tilde{\gamma}^U = 1) \geq \bar{\Delta}.
\]

(The lower bound of the unbiased analyst’s second period precision effect, \( \bar{\Delta} \), is derived in the Appendix). Therefore, if information-gathering costs are less than \( \bar{\Delta} \), the unbiased analyst will choose to acquire information, consistent with the investor’s conjecture.

For the biased analyst, the benefit from acquiring information depends on the level of future concerns because her optimal communication strategy varies with \( x \). Similar arguments as in Proposition 1 show that, if the investor conjectures that \( \tilde{\gamma}^U = 1 > \tilde{\gamma}^B = \bar{\gamma} \), then the investor will favorably update his belief about the analyst’s type when the report is consistent with the realized state. Hence the biased analyst will have greater reputational incentive to tell the truth when her precision is higher. At the same time, the biased analyst’s current incentive to report 1 is independent of her precision. As a result, having a higher precision would make the biased analyst more likely to report truthfully. The following three future concerns scenarios are mutually exclusive and commonly exhaustive:

- **Case I:** For future concerns \((1/x)\) sufficiently small, the biased analyst always reports 1 if she acquires information. *A fortiori* she will then also always report 1 if she has the default precision. In this case, the biased analyst’s precision becomes immaterial because her reporting strategies in the continuation game are independent of her precision and the investor’s conjecture.\(^\text{18}\)

\(^{19}\)Note that \( \tilde{\gamma}^B \) does not affect \( V^J(\cdot) \) because the biased analyst always reports 1 in the second period, independent of her precision. For the same reason, holding reputation \( \lambda_2 \) constant, \( V^B(\cdot) \) is also independent of her actual precision \( \gamma^B \). Hence, later I suppress the argument \( \gamma^B \) in \( V^B(\cdot) \).
signals. Hence, the biased analyst’s benefit from acquiring information in this case is zero, that is, $\Delta^B_I = 0$.

- Case II: For future concerns sufficiently important, i.e., $x < \bar{z}$, by Proposition 1, the biased analyst with the default precision will report her signal truthfully. Since having a higher precision makes her more likely to tell the truth, the biased analyst will also report her signal truthfully if she becomes better informed. That is, having a higher precision does not affect the biased analyst’s communication strategy, which in turn leaves her first period payoff unaffected (since the biased analyst’s first period payoff only depends on her report).\(^{20}\) Thus, by acquiring information, the biased analyst only benefits from the reputation enhancement. Therefore:\(^{21}\)

$$
\Delta^B_{II} = \frac{1}{2} (1 - \bar{\gamma}) \{ V^B(\Lambda(0, 0|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) - V^B(\Lambda(1, 0|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) \} \\
+ \frac{1}{2} (1 - \bar{\gamma}) \{ V^B(\Lambda(1, 1|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) - V^B(\Lambda(0, 1|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) \}.
$$

(To be precise, I am adding the previously suppressed $x$ as an argument in the $\Lambda(\cdot)$ function. Note that $\Lambda(\cdot)$ depends on the investor’s conjecture about the analyst’s communication strategy, which, in equilibrium, varies with $x$). By increasing her precision from $\bar{\gamma}$ to 1, the biased analyst increases the probability of issuing correct reports by $1 - \bar{\gamma}$; $\Delta^B_{II}$ captures the attendant gains from enhanced reputation.

- Case III: For intermediate levels of future concerns (see Appendix for details), the biased analyst tells the truth if perfectly informed, but lies when observing 0 with the default precision. By revealed preference, when the biased analyst has the default precision, her utility under such misreporting strategy is (weakly) greater than her utility under the truth-telling strategy. Hence the biased analyst’s benefit from acquiring information in this case is less than her benefit if the truth-telling strategy is implemented for both precision levels. Therefore,

$$
\Delta^B_{III} \leq \frac{1}{2} (1 - \bar{\gamma}) \{ V^B(\Lambda(0, 0|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) - V^B(\Lambda(1, 0|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) \} \\
+ \frac{1}{2} (1 - \bar{\gamma}) \{ V^B(\Lambda(1, 1|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) - V^B(\Lambda(0, 1|1, \bar{\gamma}, x), \bar{\gamma}^U = 1) \}.
$$

\(^{20}\)Note that the ex ante probability for the analyst to observe 0 and 1 is always $\frac{1}{2}$ irrespective of her precision.

\(^{21}\)I suppress the argument $\gamma^B$ in $V^B(\cdot)$ (see footnote 19).
To summarize the three cases, for any level of future concerns, the benefit from acquiring information for the biased analyst is bounded from above as follows:

$$\Delta^B \leq \frac{1}{2}(1 - \bar{\gamma})\{V^B(\Lambda(0,0|1,\bar{\gamma},x),\tilde{\gamma}^U = 1) - V^B(\Lambda(1,0|1,\bar{\gamma},x),\tilde{\gamma}^U = 1)\}$$

$$+ \frac{1}{2}(1 - \bar{\gamma})\{V^B(\Lambda(1,1|1,\bar{\gamma},x),\tilde{\gamma}^U = 1) - V^B(\Lambda(0,1|1,\bar{\gamma},x),\tilde{\gamma}^U = 1)\}$$

$$< (1 - \bar{\gamma})[V^B(\lambda_2 = 1,\tilde{\gamma}^U = 1) - V^B(\lambda_2 = 0,\tilde{\gamma}^U = 1)]$$

$$\equiv \Delta$$

Therefore, if $c > \Delta$, the biased analyst will choose to keep her default precision, consistent with the investor’s conjecture.

The key to understanding why the unbiased analyst benefits more from acquiring information than the biased analyst is to note that the investor’s decision in the second period is more sensitive to the analyst’s report than to her reputation. Figure 2 demonstrates this point. If the investor receives report 0, he knows for sure that this report is coming from an unbiased analyst and will choose $a_2(m_2 = 0, \lambda_2) = 1 - \gamma^U$. If the investor receives report 1, his decision will depend on the analyst’s reputation $\lambda_2$. Clearly $a_2(m_2 = 1, \lambda_2 = 0) = \frac{1}{2}$ and $a_2(m_2 = 1, \lambda_2 = 1) = \gamma^U$. Therefore, the maximum effect of differential reputation levels on the investor’s decision is $\gamma^U - \frac{1}{2}$, while the minimum effect of differential reports on the investor’s decision is $\gamma^U - \frac{1}{2}$. That is, the investor’s decision in the second period responds more sensitively to the analyst’s report than to her reputation.

Proposition 2 provides a sufficient condition for the existence of an informative equi-
librium for all levels of future concerns. If future concerns are sufficiently important, then this sufficient condition can be relaxed, as the following Corollary shows. To that end, define \( c^o = \frac{1}{2}(1 - \bar{\gamma})[\frac{1}{2} - \frac{2\bar{\gamma}^2(1-\lambda)^2}{(\lambda + 2\bar{\gamma}(1-\lambda))^2}] \). Note that \( c^o < \Delta < \bar{\Delta} \).

**Corollary 1** If \( c^o < c \leq \bar{\Delta} \) and \( x < x^* \), then there exists an informative equilibrium, in which only the unbiased analyst acquires information and communication in each period is informative. Specifically, both types of analysts report truthfully in the first period. In the second period, the unbiased analyst reports truthfully and the biased analyst reports 1 all the time.

Corollary 1 deals with a special case of Proposition 2, so the key arguments for Proposition 2 carry over here. If \( x < x^* \), then the benefit from acquiring information for the biased analyst is derived as above, that is,

\[
\Delta^B_{II} = \frac{1}{2}(1 - \bar{\gamma})\{V^B(\Lambda(0,0|1,\bar{\gamma},x),\bar{\gamma}^U = 1) - V^B(\Lambda(0,1|1,\bar{\gamma},x),\bar{\gamma}^U = 1)\} \\
+ \frac{1}{2}(1 - \bar{\gamma})\{V^B(\Lambda(1,1|1,\bar{\gamma},x),\bar{\gamma}^U = 1) - V^B(\Lambda(0,1|1,\bar{\gamma},x),\bar{\gamma}^U = 1)\} \\
= \frac{1}{2}(1 - \bar{\gamma})[\frac{1}{2} - \frac{2\bar{\gamma}^2(1-\lambda)^2}{(\lambda + 2\bar{\gamma}(1-\lambda))^2}] \\
\equiv c^o.
\] (7)

Hence, if \( x < x^* \) and \( c > c^o \), the biased analyst will choose to keep the default precision. On the other hand, the benefit from acquiring information for the unbiased analyst is \( \Delta^U > \bar{\Delta} \), hence the unbiased analyst will choose to acquire information.

I now address the remaining case of small information-gathering costs:

**Proposition 3** Suppose \( \bar{\gamma} \geq 0.75 \). For information-gathering costs sufficiently small and future concerns sufficiently important, i.e., \( c < c^o \) and \( x < \min\{x^o, x^*\} \), there exists a unique informative equilibrium in which only the unbiased analyst acquires information, first period communication is babbling and second period communication is informative.

If analysts’ information-gathering costs are small, then the unbiased analyst is always better off acquiring information even by the second period precision effect alone. Given that the unbiased analyst acquires information, first period communication has to be babbling for future concerns sufficiently important. Otherwise, the biased analyst would also choose to acquire information for reputation building purposes (since \( c < c^o = \Delta^B_{II} \)).
given that both analysts have the same (improved) precision, first period communication has to be babbling for $x < min\{x^0, \bar{x}\} \leq x^0$ (by Lemma 1).\textsuperscript{22} Anticipating this, the biased analyst has no incentive to acquire information because her reporting strategy in each period is independent of her signal. Hence, the biased analyst will keep the default precision.

At first glance, Proposition 3 seems to conflict with Proposition 1 since the latter states that first period informative communication always obtains (for any level of future concerns) if the unbiased analyst is better informed. However, recall that Proposition 1 speaks to the case where analysts’ precision is common knowledge. When analysts’ precision choices are unobservable, then one has to take into consideration possible deviations not only regarding analysts’ reporting strategies, but also regarding their precision choices when analyzing whether first period informative communication can be sustained in equilibrium.

To conclude, for the most interesting case where future concerns are important, I find that if information-gathering costs are sufficiently small, only the unbiased analyst acquires information and first period communication is uninformative; if information-gathering costs are moderate, again only the unbiased analyst acquires information but first period communication becomes informative; if information-gathering costs are high, neither analyst acquires information and first period communication reverts to babbling.

5 Policy Implications

In general, analysts’ information-gathering costs are influenced by many factors: access to management inside information, complexity of industry knowledge, transparency of financial disclosure, etc. All else equal, the more transparent the financial disclosure, the lower the analysts’ information-gathering costs. Proponents of increased transparency usually base their arguments on the conventional wisdom that higher transparency would increase the welfare of financially unsophisticated investors. However, it has been argued that regulatory acts requiring increased transparency (e.g., FAS 131 segment reporting) impose significant “implementation costs” on firms and hence the social welfare consequences of such regulatory acts are up to debate. The following analysis points out that even without

\textsuperscript{22}The threshold $x^0$ is given in the paragraph following Lemma 1.
such implementation costs, greater transparency may decrease social welfare by reducing
the informativeness of communication between analysts and investors.

**Corollary 2** Suppose $\gamma \geq 0.75$. Consider $c_1 = c^e - \delta$ and $c_2 = c^e + \varepsilon$, then for future concerns sufficiently important, i.e., $x < \min\{x^e, \bar{x}\}$, when $\delta \to 0$ and $\varepsilon \to 0$, decreasing analysts’ information-gathering costs from $c_2$ to $c_1$ reduces social welfare.

Consider the case where future concerns are important. Then, Corollary 1 has shown that for moderate information-gathering costs, there exists an informative equilibrium in which only the unbiased analyst acquires information and both analysts tell the truth in the first period. In contrast, by Proposition 3, given a regulatory condition on $\gamma$, communication in the first period takes the form of babbling if the analyst’s costs of acquiring information are small. That is, if information-gathering costs decrease from $c_2$ to $c_1$, communication in the first period changes from informative to babbling.

First period informative communication not only enables the investor to make better concurrent decisions, but also facilitates learning about the analyst’s type, which leads to better decision making in the second period. Similarly, the unbiased analyst, who internalizes the investor’s payoff, is also better off under informative communication given that the additional information-gathering costs are negligible as postulated in Corollary 2. In contrast, informative communication leaves the biased analyst worse off. To see this, note that the biased analyst’s payoff in the first period, $-(a_1(\cdot) - 1)^2$, is independent of the state and increasing and concave in the investor’s action. If first period communication is informative, then the investor’s investment decision will be contingent non-trivially on the analyst’s report; given the concavity of the biased analyst’s first period payoff, this will make her worse off for the first period (by Jensen’s inequality). In addition, when first period communication is informative, the biased analyst can only partially pool with the unbiased analyst (with probability $\gamma$), which reduces her payoff for the second period (recall that babbling amounts to complete pooling of analysts’ types).

Given the countervailing effects of first period communication on the various players’ payoffs, a tradeoff arises. Corollary 2 evaluates this tradeoff, with social welfare $W(\cdot)$ defined as the investor’s ex ante expected utility plus the analyst’s ex ante expected utility (before she learns her type). I show in the proof of Corollary 2 that:

$$W(c = c_2) - W(c = c_1) = \frac{1}{2}x\lambda(a_1(1) - a_1(0)) - \lambda(c_2 - c_1).$$
Figure 3: Social welfare as a function of $c$

For this figure, the parameter values are $x = 0.05$, $\bar{\gamma} = 0.8$ and $\lambda = 0.5$. 
The first term captures the net effect of the investor making informative decisions, whereas the second term represents the difference in analysts’ information-gathering costs. Clearly, the first term is positive, and the second term approaches zero when the cost difference is negligible. Hence, social welfare is reduced when information-gathering costs decrease from $c_2$ to $c_1$ (see Figure 3).

Fischer and Stocken (2010) derive a related result. They find that in some circumstances, more precise public information may “crowd out” analysts’ private information, making the investor strictly worse off. My analysis shows that in the presence of reputation formation, increased transparency may sometimes impede communication between analysts and investors, and hence may have negative social welfare consequences.

6 Empirical Implications

This paper generates several empirical predictions consistent with extant empirical studies. First, the model suggests that analysts with better reputation have greater impact on investors’ decisions, in line with Stickel (1992), Park and Stice (2000) and Jackson (2005). Second, higher reputation analysts have better future performance. This, too, is consistent with Stickel (1992) and Desai, Liang and Singh (2000), among others. Third, the model suggests that in general analysts’ reputation is increasing in their forecast accuracy, consistent with Chen, Francis, and Jiang (2005) and Jackson (2005). Fourth, on average, analysts’ reports are informative (e.g., Dimson and Marsh (1984), Womack (1996)), but optimistic (e.g., O’Brien (1988), Lys and Sohn (1990), Brown (1993), Dugar and Nathan (1995)). Fifth, if one interprets unbiased analysts as unaffiliated and biased analysts as affiliated—a reasonable link at least before the Global Settlement—this paper predicts that affiliated analysts issue more optimistic reports than unaffiliated analysts, which is again confirmed by most empirical findings (e.g., Dugar and Nathan (1995), Lin and McNichols (1998), Michaely and Womack (1999)).

The model also generates new empirical implications. First, Proposition 1 suggests that as future concerns increase in importance, the average optimism in analysts’ report decreases. NYSE Rule 472(j)(2) encourages analysts to make their track record more

---

23In these studies, analyst reputation is with regard to the precision, not the bias levels. However, these two different dichotomies converge as unbiased analysts have higher precision in equilibrium when information-gathering costs are moderate. Hence, in equilibrium analyst reputation conveys information both about the bias levels and the precision.
transparent, which in turn increases the ability of investors (especially small ones) to form an opinion about analysts’ type. Prior studies (e.g., Barber, Lehavy, McNichols, and Trueman (2006), Kadan, Madureira, Wang, and Zach (2009)) document a decrease in analyst optimism subsequent to the 2002-2003 regulatory changes, but attribute this mainly to the decline in investment banking conflict of interest. My model suggests that increased reputational concerns due to NYSE Rule 472(j)(2) might be a confounding factor.

Furthermore, Proposition 2 suggests that in general unbiased analysts tend to have more precise information and are more likely to report truthfully. Hence, from an ex post perspective, my model predicts an endogenous association between analysts’ forecast accuracy and bias. More specifically, analysts with higher forecast accuracy are less optimistic. This prediction is confirmed by Conroy and Harris (1995). However, Lim (2001) and several other studies (e.g., Chen and Matsumoto (2006)) hold the contrary view that more optimistic analysts tend to be more accurate because of privileged access to management inside information. Regulation FD aims to prohibit selective disclosures with the goal of creating a more even playing field among analysts. I therefore expect that my model speaks more to the post-Reg FD regime. Furthermore, my results imply that if biased analysts have a significant informational advantage over unbiased analysts, then communication may become uninformative. Hence Reg FD has the additional benefit of fostering communication by limiting biased analysts’ informational advantage.

Somewhat surprisingly, Proposition 3 suggests that if learning more about firms comes at a low cost to analysts (e.g., due to high disclosure standards) and future concerns are weighed heavily, analysts’ recommendations will be less informative.

7 Conclusion

This paper investigates how the presence of reputation formation affects analysts’ strategic communication with investors when analysts can acquire information. If information-gathering costs are moderate, only unbiased analysts acquire information; as a result, informative communication can be sustained in equilibrium. That is, analysts’ communication in the reputation formation stage conveys information both about their types (the bias levels) and their precision (given the endogenous association between the analysts’ types and precision). On the other hand, if information-gathering costs are small and analysts care a lot about the future, communication becomes uninformative. Hence, efforts
aimed at reducing information-gathering costs for analysts may have adverse effects on social welfare.

Certain key features of the model warrant further discussion. For example, I assume that analysts’ precision is perfectly correlated over time. In the extreme opposite case of zero serial correlation (i.e., acquiring information only increases analysts’ precision in the first period), neither analyst will acquire information regardless of information-gathering costs if analysts care a lot about future transactions. In practice, however, some elements of analysts’ information sets are clearly persistent (e.g., industry knowledge, the ability to analyze financial statement). My qualitative findings (assuming perfect correlation) will continue to hold when the correlation is sufficiently large.

Finally, in this paper I only consider analysts’ messages about their signals. What if one allows analysts to send multidimensional cheap talk messages? For example, analysts could send a separate message about their precision in addition to the one about their signals. Because I confine my attention to pure strategies for analysts’ information acquisition decisions, this possibility does not affect my analysis. The reason is that, when analysts are confined to pure strategies for their information acquisition choices, investors will ignore analysts’ separate cheap talk messages about their precision because their precision choices are deterministic. An interesting model extension is to allow for mixed strategies for analysts’ information acquisition decisions or to introduce noise into the analysts’ learning technology. The extent to which this extension would affect my results has yet to be explored.

24The key to understanding this result is that biased analysts benefit more from a high reputation than their unbiased peers. When future concerns are important, both types of analysts acquire information mainly for reputation building purposes. Then, whenever it is profitable for the unbiased analysts to acquire information, it is also optimal for the biased analysts to do so. By Lemma 1, first period communication then becomes uninformative. As a result, neither analyst has an incentive to become better informed.
8 Appendix

Formal definition of the equilibrium

I begin by introducing the analyst’s utility at each decision node. To that end, denote by $U_t^J(m_t, s_t, \lambda_t, \gamma^J)$ the type $J$ analyst’s utility at period $t = 1, 2$ when she reports $m_t$ and her signal is $s_t$, reputation is $\lambda_t$ and precision is $\gamma^J$. Specifically, $\lambda_1 = \lambda$ and $\lambda_2 = \Lambda(m_1, w_1)$. Analyst $J$’s utility at the information acquisition stage is denoted by $U_0^J(\gamma^J)$.\(^{25}\)

Given the investor’s second period optimal decision rule $a_2(m_2, \Lambda)$, type $J$ analyst’s second period utility is:

$$
U_2^U(m_2, s_2, \Lambda, \gamma^U) = -\gamma^U(a_2(m_2, \Lambda) - s_2)^2 - (1 - \gamma^U)[a_2(m_2, \Lambda) - (1 - s_2)]^2,
$$

$$
U_2^B(m_2, s_2, \Lambda, \gamma^B) = -(a_2(m_2, \Lambda) - 1)^2.
$$

Denote by $\sigma_t^j(s_t, \gamma^J, \lambda_t)$ the type $J$ analyst’s optimal communication strategy at period $t$ upon observing signal $s_t$ when her precision is $\gamma^J$ and reputation is $\lambda_t$. Then,

$$
\sigma_2^J(s_2, \gamma^J, \Lambda) \in \arg\max_{\sigma_2^J \in [0, 1]} \sigma_2^J U_2^J(1, s_2, \Lambda, \gamma^J) + (1 - \sigma_2^J)U_2^J(0, s_2, \Lambda, \gamma^J).
$$

Hence, type $J$ analyst’s second period expected utility (anticipating that she will communicate optimally) is\(^{26}\)

$$
V_J^J(\Lambda, \gamma^J) = \frac{1}{2} \sum_{s_2 = 0, 1} \sigma_2^J(s_2, \gamma^J, \Lambda) U_2^J(1, s_2, \Lambda, \gamma^J) + (1 - \sigma_2^J(s_2, \gamma^J, \Lambda)) U_2^J(0, s_2, \Lambda, \gamma^J).
$$

Given the investor’s first period optimal decision rule $a_1(m_1, \lambda)$, type $J$ analyst’s 1st period utility is then given by:

$$
U_1^U(m_1, s_1, \lambda, \gamma^U) = -\gamma^U(a_1(m_1, \lambda) - s_1)^2 - (1 - \gamma^U)[a_1(m_1, \lambda) - (1 - s_1)]^2
\quad + \gamma^U V^U(\Lambda(m_1, s_1), \gamma^U) + (1 - \gamma^U) V^U(\Lambda(m_1, 1 - s_1), \gamma^U),
$$

$$
U_1^B(m_1, s_1, \lambda, \gamma^B) = -(a_1(m_1, \lambda) - 1)^2 + \gamma^B V^B(\Lambda(m_1, s_1), \gamma^B) + (1 - \gamma^B) V^B(\Lambda(m_1, 1 - s_1), \gamma^B).
$$

Then,

$$
\sigma_1^J(s_1, \gamma^J, \lambda) \in \arg\max_{\sigma_1^J \in [0, 1]} \sigma_1^J U_1^J(1, s_1, \lambda, \gamma^J) + (1 - \sigma_1^J)U_1^J(0, s_1, \lambda, \gamma^J).
$$

\(^{25}\)To save on notation, I suppress the functional dependence of the players’ utilities and strategies on their conjectures about their counterparties’ actions.

\(^{26}\)Again, here I suppress the dependence of $V_J^J(\cdot)$ on $\tilde{\gamma}$. 

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Thus, type $J$ analyst’s utility at the information acquisition stage is given by

$$U_J^J(\gamma^J) = \frac{1}{2} \left\{ \sum_{s_1=0,1} \sigma_t^J(s_1, \gamma^J, \lambda) U_t^J(1, s_1, \lambda, \gamma^J) + (1 - \sigma_t^J(s_1, \gamma^J, \lambda)) U_t^J(0, s_1, \lambda, \gamma^J) \right\} - C(\gamma^J).$$

Now I am in a position to define the equilibrium of the game formally.

**Definition 3** A Perfect Bayesian Nash equilibrium of the game is a strategy-belief profile $(\gamma_U^*, \gamma_B^*, \sigma_U^J(\cdot), \sigma_B^J(\cdot), a_t(\cdot), \Gamma_1(\cdot), \Lambda(\cdot))$ satisfying the following properties:

1. $$\sigma_t^J(s_t, \gamma^J, \lambda_t) \in \arg\max_{\sigma_t^J \in [0,1]} \sigma_t^J U_t^J(1, s_t, \lambda_t, \gamma^J) + (1 - \sigma_t^J) U_t^J(0, s_t, \lambda_t, \gamma^J).$$

2. $$a_t(m_t, \lambda_t) \in \arg\max_{a_t \in R} - \Gamma_t(m_t, \lambda_t)(a_t - 1)^2 - (1 - \Gamma_t(m_t, \lambda_t))a_t^2.$$

3. $$\gamma^J \in \arg\max_{\gamma^J \in [0,1]} U_0^J(\gamma^J).$$

4. The state and type inference functions, $\Gamma_1(m_1, \lambda)$, $\Gamma_2(m_2, \Lambda)$ and $\Lambda(\cdot)$, are derived from the analyst’s equilibrium strategy according to inference rules (1) and (2).

Specifically,

$$\phi_t^J(1|w_t) = \gamma^J \sigma_t^J(w_t, \gamma^J, \lambda_t) + (1 - \gamma^J) \sigma_t^J(1 - w_t, \gamma^J, \lambda_t).$$

When the posteriors are undefined according to Bayes rule, I adopt the convention that the investor sticks to his priors.

The analysis in Section 3.1 shows that the analyst’s second period optimal reporting strategy is independent of her reputation $\lambda_2$. Furthermore, for the first period, $\lambda_1(= \lambda)$ is a constant, hence I drop the argument $\lambda_1$ in $\sigma_t^J(\cdot)$ in the main text.

**Proof of Lemma 1**

In order to prove Lemma 1, I first extend the insights from Morris (2001) and characterize the properties of first period informative communication for exogenous analyst’s precision $\gamma = (\gamma_U, \gamma_B)$. Then I show that if $\gamma_B \geq \gamma_U$, when future concerns are sufficiently important, i.e., $x$ is sufficiently small, no informative (first period) communication obtains.
Let me introduce the notations under exogenous and commonly known \( \gamma = (\gamma^U, \gamma^B) \): \( \sigma_t^J(s_t|\gamma) \) is the probability of type \( J \) analyst reporting 1 in period \( t \) when her signal is \( s_t \); \( a_t(m_t, \lambda_t, \gamma) \) is the investor’s action in period \( t \) when he receives message \( m_t \) and his belief of analyst being unbiased is \( \lambda_t \); \( \Gamma_t(m_t, \lambda_t, \gamma) \) states the investor’s inference of the actual state being 1 in period \( t \); \( \Lambda(m_1, w_1|\gamma) \) is the investor’s belief of the analyst being unbiased if message \( m_1 \) is received and state \( w_1 \) is realized. \( V^J(\Lambda(\cdot), \gamma^U) \) is the type \( J \) analyst’s second period expected utility when she has reputation \( \Lambda(\cdot) \). Note that \( V^J(\cdot) \) is independent of \( \gamma^B \). The reason is that the biased analyst always reports 1 in the second period and hence her precision has no effect on the investor’s action. The analyst’s current incentives and reputational incentives are defined in (5) and (6) in the main text. To save on notation, I suppress the argument \( \lambda \) in \( \Gamma_1(\cdot) \) and \( a_1(\cdot) \).

**Claim 1** When the unbiased analyst is perfectly informed, i.e., \( \gamma^U = 1 \), any informative communication in the first period has to satisfy the following properties:

1. **The investor updates favorably his belief about the analyst’s type when the analyst’s report is consistent with the realized state.** More specifically,
   \[
   \Lambda(1, 1|\gamma^U = 1, \gamma^B) \geq \Lambda(0, 1|\gamma^U = 1, \gamma^B) \quad \text{and} \quad \Lambda(0, 0|\gamma^U = 1, \gamma^B) \geq \Lambda(1, 0|\gamma^U = 1, \gamma^B),
   \]
   and at least one of the inequalities is strict;

2. **The unbiased analyst always reports truthfully;**

3. **The biased analyst always reports 1 when she observes signal 1.**

The above properties hold for generic \( \gamma^B \). I prove the result in ten steps. Each step identifies a property that must hold for first period informative communication when \( \gamma^U = 1 \). The first four steps hold for generic \( \gamma = (\gamma^U, \gamma^B) \).

**Property 1.** \( \Lambda(0, 0|\gamma) \geq \Lambda(1, 0|\gamma) \).

Prove by contradiction. Suppose \( \Lambda(0, 0|\gamma) < \Lambda(1, 0|\gamma) \), there are 2 subcases:

(i) \( \Lambda(0, 0|\gamma) < \Lambda(1, 0|\gamma) \) and \( \Lambda(1, 1|\gamma) > \Lambda(0, 1|\gamma) \).

Now \( \Pi^{B}(s_1|\gamma) < 0 \) and \( \Pi^{B}(s_1|\gamma) \geq 0 \) for \( s_1 = 0, 1 \). Therefore \( \sigma^B_t(0|\gamma) = \sigma^B_t(1|\gamma) = 1 \). Then if \( \sigma^U_t(0|\gamma) = \sigma^U_t(1|\gamma) = 1 \), the communication in the first period is uninformative, which is a contradiction. If \( \sigma^U_t(0|\gamma) \neq 1 \) or \( \sigma^U_t(1|\gamma) \neq 1 \), then \( \Lambda(0, 1|\gamma) = 1 \) or \( \Lambda(0, 0|\gamma) = 1 \), contradicting “\( \Lambda(0, 0|\gamma) < \Lambda(1, 0|\gamma) \) and \( \Lambda(1, 1|\gamma) > \Lambda(0, 1|\gamma) \)”.

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(ii) $\Lambda(0,0|\gamma) < \Lambda(1,0|\gamma)$ and $\Lambda(1,1|\gamma) \leq \Lambda(0,1|\gamma)$.

In this case, $\Pi_B^B(1|\gamma) > \Pi_B^B(0|\gamma)$, which, together with the fact that $\Pi_B^B(1|\gamma) = \Pi_B^B(0|\gamma)$, implies that either $\sigma^B(1|\gamma) = 0$, or $\sigma^B(0|\gamma) = 1$. Then,

$$\phi^B_1(1|1) = \gamma^B \sigma^B_1(1|1) + (1 - \gamma^B)\sigma^B_1(0|\gamma) \leq (1 - \gamma^B)\sigma^B_1(1|1) + \gamma^B \sigma^B_1(0|\gamma) = \phi^B_1(1|0).$$

On the other hand, by the definition of $\Lambda$ in (2), $\Lambda(0,0|\gamma) < \Lambda(1,0|\gamma) \Rightarrow \phi^U_1(1|0) > \phi^B_1(1|0)$, and $\Lambda(1,1|\gamma) \leq \Lambda(0,1|\gamma) \Rightarrow \phi^U_1(1|1) \leq \phi^B_1(1|1)$. Therefore, $\phi^U_1(1|1) \leq \phi^B_1(1|1) \leq \phi^B_1(1|0) < \phi^U_1(1|0)$. Thus, by (1), $a_1(1,\gamma) = \Gamma_1(1,\gamma) < \Gamma_1(0,\gamma) = a_1(0,\gamma)$. A contradiction (recall that $a_1(1,\gamma) \geq a_1(0,\gamma)$ is assumed without loss of generality).

To conclude, $\Lambda(0,0|\gamma) < \Lambda(1,0|\gamma)$ is impossible for informative communication. Therefore $\Lambda(0,0|\gamma) \geq \Lambda(1,0|\gamma)$.

**Property 2.** $a_1(1,\gamma) = \Gamma_1(1,\gamma) > \Gamma_1(0,\gamma) = a_1(0,\gamma)$.

Given property 1, $\Lambda(0,0|\gamma) \geq \Lambda(1,0|\gamma)$, there are two subcases. The idea is to show that $\Gamma_1(1,\gamma) > \Gamma_1(0,\gamma)$ holds for both subcases.

(i) $\Lambda(0,0|\gamma) \geq \Lambda(1,0|\gamma)$ and $\Lambda(0,1|\gamma) \geq \Lambda(1,1|\gamma)$.

First, I claim that at least one of the inequalities is strict. Suppose not, that is, both inequalities hold with equality. If $\Gamma_1(1,\gamma) = \Gamma_1(0,\gamma)$, the communication is noninformative, which is a contradiction. If $\Gamma_1(1,\gamma) > \Gamma_1(0,\gamma)$, then the biased analyst will always report 1 since there are no reputational consequences. Thus, the investor will know for sure that the analyst is unbiased if he receives message 0. Hence, $\Lambda(0,0|\gamma) = 1$, which implies that $\Lambda(1,0|\gamma) = 1$ given that $\Lambda(0,0|\gamma) = \Lambda(1,0|\gamma)$. Obviously, $\Lambda(1,0|\gamma) = 1$ is contradicting $\sigma^B(0|\gamma) = \sigma^B(1|\gamma) = 1$. Therefore, at least one of the inequalities is strict.

Suppose $\Gamma_1(1,\gamma) = \Gamma_1(0,\gamma)$. By (1),

$$\Gamma_1(1,\gamma) = \Gamma_1(0,\gamma) \iff \frac{\lambda \phi^U_1(1|0) + (1 - \lambda)\phi^B_1(1|0)}{\lambda \phi^U_1(1|1) + (1 - \lambda)\phi^B_1(1|1)} = \frac{\lambda \phi^U_1(0|0) + (1 - \lambda)\phi^B_1(0|0)}{\lambda \phi^U_1(0|1) + (1 - \lambda)\phi^B_1(0|1)} \iff \frac{\lambda \phi^U_1(1|0) + (1 - \lambda)\phi^B_1(1|0)}{\lambda \phi^U_1(1|1) + (1 - \lambda)\phi^B_1(1|1)} = 1 \iff \lambda(2\gamma^U - 1)[\sigma^U_1(1|\gamma) - \sigma^U_1(0|\gamma)] + (1 - \lambda)(2\gamma^B - 1)[\sigma^B_1(1|\gamma) - \sigma^B_1(0|\gamma)] = 0.$$

As shown above at least one of the two inequalities, $\Lambda(0,0|\gamma) \geq \Lambda(1,0|\gamma)$ and $\Lambda(0,1|\gamma) \geq \Lambda(1,1|\gamma)$, is strict. Suppose $\Lambda(0,0|\gamma) > \Lambda(1,0|\gamma)$. Then $\Pi_B^B(0|\gamma) > 0$ for $J = B, U$. 31
In addition, $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma) \Rightarrow \Pi^U_C(0|\gamma) = 0$ for each $J = B, U$. Thus $\sigma^U_I(0|\gamma) = \sigma^B_I(0|\gamma) = 0$, which, by (8), implies that, for $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma)$ to hold, it must be true that $\sigma^U_I(1|\gamma) = \sigma^B_I(1|\gamma) = 0$. Then the communication is noninformative. A contradiction. Analogously, if $\Lambda(0, 1|\gamma) > \Lambda(1, 1|\gamma)$, then $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma)$, which also leads to a contradiction. Therefore, $\Gamma_1(1, \gamma) > \Gamma_1(0, \gamma)$ has to hold in this subcase.

(ii) $\Lambda(0, 0|\gamma) \geq \Lambda(1, 0|\gamma)$ and $\Lambda(0, 1|\gamma) < \Lambda(1, 1|\gamma)$.

In this case, $\Pi^B_R(0|\gamma) > \Pi^B_R(1|\gamma)$ and $\Pi^U_R(0|\gamma) > \Pi^U_R(1|\gamma)$. Suppose $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma)$. Then $\Pi^U_C(s_1|\gamma) = 0$ for each $J = B, U$ and $s_1 = 0, 1$. Therefore, for $J = B, U$, either $\sigma^U_I(0|\gamma) = 0$, or $\sigma^U_I(1|\gamma) = 1$. Hence, $\sigma^U_I(1|\gamma) \geq \sigma^U_I(0|\gamma)$. As a result, by (8), $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma)$ holds only when $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma)$ for $J = B, U$. There are four scenarios then: (a) if $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma) = \sigma^B_I(1|\gamma) = \sigma^B_I(0|\gamma) = 1$, then the communication is noninformative. A contradiction; (b) if $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma) = \sigma^B_I(1|\gamma) = \sigma^B_I(0|\gamma) = 0$, again the communication is noninformative. A contradiction; (c) if $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma) = 1$ and $\sigma^B_I(1|\gamma) = \sigma^B_I(0|\gamma) = 0$, then $\Lambda(0, 0|\gamma) = 0$ and $\Lambda(0, 1|\gamma) = 1$, contradicting $\Lambda(0, 0|\gamma) \geq \Lambda(1, 0|\gamma)$; (d) if $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma) = 0$ and $\sigma^B_I(1|\gamma) = \sigma^B_I(0|\gamma) = 1$, then $\Lambda(1, 1|\gamma) = 0$ and $\Lambda(0, 1|\gamma) = 1$, contradicting $\Lambda(0, 1|\gamma) < \Lambda(1, 1|\gamma)$. Therefore, $\Gamma_1(1, \gamma) = \Gamma_1(0, \gamma)$ does not hold.

To summarize, in any first period informative communication, $\Gamma_1(1, \gamma) > \Gamma_1(0, \gamma)$ has to hold.

**Property 3.** $\sigma^U_I(0|\gamma) = 0 \Rightarrow \sigma^U_I(1|\gamma) > 0$.

Prove by contradiction. Suppose $\sigma^U_I(0|\gamma) = \sigma^U_I(1|\gamma) = 0$, then for $\Gamma_1(1, \gamma) > \Gamma_1(0, \gamma)$ to hold, it must be true that $\sigma^U_I(1|\gamma) > \sigma^B_I(0|\gamma)$. Thus, $\phi^B_I(1|1) > \phi^B_I(1|0)$, which directly leads to $\phi^B_I(1|1) < \phi^B_I(0|0)$. On the other hand, $\sigma^U_I(0|\gamma) = \sigma^U_I(1|\gamma) = 0$ implies that $\phi^U_I(0|1) = \phi^U_I(0|0) = 1$. Hence, $\phi^B_I(0|0) < \phi^U_I(0|0)$, which means $\Lambda(0, 1|\gamma) > \Lambda(0, 0|\gamma)$.

In addition, $\sigma^U_I(1|\gamma) = \sigma^U_I(0|\gamma) = 0$ implies $\Lambda(1, 1|\gamma) = \Lambda(1, 0|\gamma) = 0$. Therefore, $\Pi^B_R(1|\gamma) > \Pi^B_R(0|\gamma)$. Recall that $\Pi^B_R(1|\gamma) = \Pi^B_R(0|\gamma)$, hence $\sigma^B_I(1|\gamma) \leq \sigma^B_I(0|\gamma)$ has to hold, contradicting $\sigma^B_I(1|\gamma) > \sigma^B_I(0|\gamma)$. Therefore, $\sigma^U_I(0|\gamma) = 0$ and $\sigma^U_I(1|\gamma) = 0$ cannot hold at the same time. Hence, $\sigma^U_I(0|\gamma) = 0 \Rightarrow \sigma^U_I(1|\gamma) > 0$.

**Property 4.** $\sigma^B_I(0|\gamma) = 0 \Rightarrow \sigma^B_I(0|\gamma) \leq \sigma^B_I(1|\gamma)$.

Suppose $\sigma^B_I(0|\gamma) = 0$ and $\sigma^B_I(0|\gamma) > \sigma^B_I(1|\gamma)$. Then, by Property 3, $\sigma^U_I(1|\gamma) > 0$. Thus, $\phi^U_I(1|1) > \phi^U_I(1|0)$ and $\phi^U_I(0|1) < \phi^U_I(0|0)$. On the other hand, $\sigma^B_I(0|\gamma) > \sigma^B_I(1|\gamma)$ implies that $\phi^B_I(1|1) < \phi^B_I(1|0) = \phi^B_I(0|1)$ and $\phi^B_I(0|1) > \phi^B_I(0|0)$. Therefore, $\phi^B_I(0|1) < \phi^U_I(0|0) < \phi^B_I(0|1)$ and
\[\frac{\sigma^B(0|1)}{\sigma^B(1|1)} < \frac{\sigma^B(1|0)}{\sigma^B(1|0)}, \text{ which leads to } \Lambda(0,0|\gamma) > \Lambda(0,1|\gamma) \text{ and } \Lambda(1,1|\gamma) > \Lambda(1,0|\gamma). \text{ Then}
\]

\[
\Pi^B_R(1|\gamma) - \Pi^B_R(0|\gamma) = (2\gamma^B - 1)\left[V^B(\Lambda(0,1|\gamma), \gamma^U) - V^B(\Lambda(0,0|\gamma), \gamma^U) + V^B(\Lambda(1,0|\gamma), \gamma^U) - V^B(\Lambda(1,1|\gamma), \gamma^U)\right] < 0.
\]

That is, \(\Pi^B_R(1|\gamma) < \Pi^B_R(0|\gamma)\). Recall that \(\Pi^B_R(1|\gamma) = \Pi^B_C(0|\gamma)\), so \(\sigma^B(0|\gamma) \leq \sigma^B(1|\gamma)\), contradicting \(\sigma^B(0|\gamma) > \sigma^B(1|\gamma)\). Hence, \(\sigma^U(0|\gamma) = 0\) and \(\sigma^U(0|\gamma) > \sigma^B(1|\gamma)\) cannot hold at the same time. Therefore, \(\sigma^U(0|\gamma) = 0 \Rightarrow \sigma^B(0|\gamma) \leq \sigma^B(1|\gamma)\).

**Property 5.** \(\sigma^U(0|1,\gamma^B) = 0\).

Property 1, \(\Lambda(0,0|\gamma) \geq \Lambda(1,0|\gamma)\), implies that \(\Pi^U_R(0|1,\gamma^B) \geq 0\). Property 2, \(\Gamma_1(1,\gamma) > \Gamma_1(0,\gamma)\), implies that \(\Pi^U_C(0|1,\gamma^B) < 0\). Therefore \(\sigma^U(0|1,\gamma^B) = 0\).

**Property 6.** \(\sigma^U(1|1,\gamma^B) > 0\) and \(\sigma^B(0|1,\gamma^B) \leq \sigma^B(1|1,\gamma^B)\).

These directly follow from Property 3, Property 4, and Property 5.

**Property 7.** \(\sigma^U(1|1,\gamma^B) = 1\).

I prove this property by contradiction. This proof involves several steps:

(i) \(\sigma^U(1|1,\gamma^B) \neq 1 \Rightarrow \Lambda(0,1|1,\gamma^B) > \Lambda(1,1|1,\gamma^B)\).

Suppose \(\sigma^U(1|1,\gamma^B) \neq 1\), then \(0 < \sigma^U(1|1,\gamma^B) < 1\), which implies \(\Pi^U_C(1|1,\gamma^B) = \Pi^U_R(1|1,\gamma^B)\). At the same time, \(\Gamma_1(1,\gamma) > \Gamma_1(0,\gamma)\) implies that \(\Pi^U_C(1|1,\gamma^B) > 0\). Therefore \(\Pi^U_R(1|1,\gamma^B) > 0\), which leads to \(\Lambda(0,1|1,\gamma^B) > \Lambda(1,1|1,\gamma^B)\).

(ii)

\begin{align*}
\Lambda(0,1|1,\gamma^B) > \Lambda(1,1|1,\gamma^B), \quad \sigma^U(1|1,\gamma^B) \neq 1 & \Rightarrow \\
& \quad \Rightarrow \begin{cases} 
\sigma^B(1|1,\gamma^B) > 0, \\
V^B(\Lambda(0,0|1,\gamma^B), 1) - V^B(\Lambda(1,0|1,\gamma^B), 1) \\
\qquad \geq V^B(\Lambda(0,1|1,\gamma^B), 1) - V^B(\Lambda(1,1|1,\gamma^B), 1).
\end{cases}
\end{align*}

By property 6, \(\sigma^B(0|1,\gamma^B) \leq \sigma^B(1|1,\gamma^B)\). This includes the following 4 cases:

(a) \(\sigma^B(0|1,\gamma^B) = \sigma^B(1|1,\gamma^B) = 0\).

In this case, \(\Lambda(1,1|1,\gamma^B) = 1\), contradicting \(\Lambda(0,1|1,\gamma^B) > \Lambda(1,1|1,\gamma^B)\). Therefore, \(\sigma^B(0|1,\gamma^B) = \sigma^B(1|1,\gamma^B) = 0\) is impossible.

(b) \(0 \leq \sigma^B(0|1,\gamma^B) < \sigma^B(1|1,\gamma^B) \leq 1\).

\(0 < \sigma^B(1|1,\gamma^B) \leq 1 \Rightarrow \Pi^B_R(1|1,\gamma^B) \leq \Pi^B_C(1|1,\gamma^B); \text{ and } 0 \leq \sigma^B(0|1,\gamma^B) < 1 \Rightarrow \Pi^B_R(0|1,\gamma^B) \geq \Pi^B_C(0|1,\gamma^B). \text{ Therefore, } \Pi^B_R(0|1,\gamma^B) \geq \Pi^B_C(0|1,\gamma^B) = \Pi^B_C(1|1,\gamma^B) \geq \Pi^B_R(1|1,\gamma^B). \text{ By the definition of } \Pi^B_R(\cdot), \Pi^B_R(0|1,\gamma^B) \geq \Pi^B_R(1|1,\gamma^B) \Rightarrow V^B(\Lambda(0,0|1,\gamma^B), 1) - V^B(\Lambda(1,0|1,\gamma^B), 1) \geq V^B(\Lambda(0,1|1,\gamma^B), 1) - V^B(\Lambda(1,1|1,\gamma^B), 1).

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(c) $0 < \sigma^B_1(0|1, \gamma^B) = \sigma^B_1(1|1, \gamma^B) < 1.$

$0 < \sigma^B_1(1|1, \gamma^B) < 1 \Rightarrow \Pi^B_R(1|1, \gamma^B) = \Pi^B_C(1|1, \gamma^B);$ and $0 < \sigma^B_1(0|1, \gamma^B) < 1 \Rightarrow \Pi^B_R(0|1, \gamma^B) = \Pi^B_C(0|1, \gamma^B).$ Therefore, $\Pi^B_R(0|1, \gamma^B) = \Pi^B_C(0|1, \gamma^B) = \Pi^B_C(1|1, \gamma^B) = \Pi^B_R(1|1, \gamma^B).$ By the definition of $\Pi^B_R(\cdot), \Pi^B_R(0|1, \gamma^B) = \Pi^B_R(1|1, \gamma^B) \Rightarrow V^B(\Lambda(0, 0|1, \gamma^B), 1) - V^B(\Lambda(1, 0|1, \gamma^B), 1) = V^B(\Lambda(0, 1|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1).

(d) $\sigma^B_1(0|1, \gamma^B) = \sigma^B_1(1|1, \gamma^B) = 1.$

In this case, it must be true that $\Lambda(0, 1|1, \gamma^B) = \Lambda(0, 0|1, \gamma^B) = 1.$ Since $\sigma^U_1(0|1, \gamma^B) = 0$ (property 5) and $0 < \sigma^U_1(1|1, \gamma^B) < 1,$ it must be true that $\Lambda(1, 0|1, \gamma^B) = 0$ and $0 < \Lambda(1, 1|1, \gamma^B) < \lambda.$ Thus, $V^B(\Lambda(0, 0|1, \gamma^B), 1) - V^B(\Lambda(1, 0|1, \gamma^B), 1) > V^B(\Lambda(0, 1|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1).

(iii)

\[
V^B(\Lambda(0, 0|1, \gamma^B), 1) - V^B(\Lambda(1, 0|1, \gamma^B), 1) \\
\geq V^B(\Lambda(0, 1|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1),
\]

$\Lambda(0, 1|1, \gamma^B) > \Lambda(1, 1|1, \gamma^B),$ $\Rightarrow \sigma^U_1(1|1, \gamma^B) = 1.$

Now, we have

\[
\Pi^B_R(1|1, \gamma^B) = \gamma^B[V^B(\Lambda(0, 1|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1)] + \left(1 - \gamma^B\right)[V^B(\Lambda(0, 0|1, \gamma^B), 1) - V^B(\Lambda(1, 0|1, \gamma^B), 1)] \\
\geq V^B(\Lambda(0, 1|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1) \\
> V^U(\Lambda(0, 1|1, \gamma^B), 1) - V^U(\Lambda(1, 1|1, \gamma^B), 1) \\
= \Pi^U_R(1|1, \gamma^B).
\]

The last inequality comes from $\Lambda(0, 1|1, \gamma^B) > \Lambda(1, 1|1, \gamma^B)$ and $\frac{\partial V^U(\lambda_2, \gamma^U)}{\partial \lambda_2} < \frac{\partial V^B(\lambda_2, \gamma^B)}{\partial \lambda_2}.$ At the same time, $\Pi^B_R(1|1, \gamma^B) = \Pi^B_C(1|1, \gamma^B).$ In addition, $\sigma^B_1(1|1, \gamma^B) > 0 \Rightarrow \Pi^B_R(1|1, \gamma^B) \leq \Pi^B_C(1|1, \gamma^B).$ Therefore, $\Pi^U_R(1|1, \gamma^B) = \Pi^B_C(1|1, \gamma^B) \geq \Pi^B_R(1|1, \gamma^B) > \Pi^U_R(1|1, \gamma^B).$ As a result, $\sigma^U_1(1|1, \gamma^B) = 1,$ which is a contradiction.

**Property 8.** $\Lambda(1, 1|1, \gamma^B) \geq \Lambda(0, 1|1, \gamma^B).$

Since $\gamma^U = 1,$ $\sigma^U_1(1|1, \gamma^B) = 1 \Rightarrow \phi^U_1(1|1) = \sigma^U_1(1|1, \gamma^B) = 1 \geq \phi^B_1(1|1).$ Therefore, $\Lambda(1, 1|1, \gamma^B) \geq \Lambda(0, 1|1, \gamma^B).

**Property 9.** $\Lambda(0, 0|1, \gamma^B) \geq \Lambda(0, 0|1, \gamma^B)$ and $\Lambda(1, 1|1, \gamma^B) \geq \Lambda(0, 1|1, \gamma^B),$ and at least one of these inequalities is strict.
By Property 1 and Property 8, \( \Lambda(0, 0|1, \gamma^B) \geq \Lambda(1, 0|1, \gamma^B) \) and \( \Lambda(1, 1|1, \gamma^B) \geq \Lambda(0, 1|1, \gamma^B) \). Suppose both hold with equality. Then \( \Gamma_1(1, \gamma) > \Gamma_1(0, \gamma) \) implies \( \sigma_1^R(1|1, \gamma^B) = \sigma_1^B(0|1, \gamma^B) = 1 \), since there is no reputational consequences and the current incentive is to report 1. Together with the fact that \( \sigma_1^U(1|1, \gamma^B) = 1 \) (Property 7) and \( \sigma_1^U(0|1, \gamma^B) = 0 \) (Property 5), \( \sigma_1^B(1|1, \gamma^B) = \sigma_1^B(0|1, \gamma^B) = 1 \) imply that \( \Lambda(0, 0|1, \gamma^B) = 1 > \Lambda(0, 0|1, \gamma^B) = 0 \), contradicting \( \Lambda(0, 0|1, \gamma^B) = \Lambda(1, 0|1, \gamma^B) \).

**Property 10.** \( \sigma_1^B(1|1, \gamma^B) = 1 \).

By Property 9 and the definition of \( \Pi_R^B(\cdot) \), \( \Pi_R^B(1|1, \gamma^B) < \Pi_R^B(0|1, \gamma^B) \), which implies that \( \sigma_1^B(0|1, \gamma^B) = 0 \) or \( \sigma_1^B(1|1, \gamma^B) = 1 \) (because \( \Pi_C^B(1|1, \gamma^B) = \Pi_C^B(0|1, \gamma^B) \)). Suppose that \( \sigma_1^B(1|1, \gamma^B) \neq 1 \), then \( \sigma_1^B(0|1, \gamma^B) = 0 \) has to hold. Thus,

\[
\phi_1^B(1|1) - \phi_1^B(0|0) = \phi_1^B(1|1) + \phi_1^B(1|0) - 1 = \sigma_1^B(1|1, \gamma^B) + \sigma_1^B(0|1, \gamma^B) - 1 = \sigma_1^B(1|1, \gamma^B) - 1 \leq 0.
\]

That is, \( \phi_1^B(1|1) \leq \phi_1^B(0|0) \). Recall that \( \sigma_1^U(1|1, \gamma^B) = 1 \) and \( \sigma_1^U(0|1, \gamma^B) = 0 \), therefore \( \Lambda(1, 0|1, \gamma^B) = \Lambda(0, 1|1, \gamma^B) = 0 \) and \( \phi_1^U(1|1) = 1 \). Hence, by \( \phi_1^B(1|1) \leq \phi_1^B(0|0) \),

\[
0 < \Lambda(0, 0|1, \gamma^B) = \frac{\lambda}{\lambda + (1 - \lambda)\phi_1^B(0|0)} \leq \frac{\lambda}{\lambda + (1 - \lambda)\phi_1^B(1|1)} = \Lambda(1, 1|1, \gamma^B).
\]

Therefore,

\[
\Pi_R^B(1|1, \gamma^B) = (2\gamma^B - 1)[V^B(\lambda_2 = 0, 1) - V^B(\Lambda(0, 0|1, \gamma^B), 1)] + \gamma^B[V^B(\Lambda(0, 0|1, \gamma^B), 1) - V^B(\Lambda(1, 1|1, \gamma^B), 1)] < 0.
\]

Also \( \Gamma_1(1, \gamma) > \Gamma_1(0, \gamma) \) imply \( \Pi_C^B(1|1, \gamma^B) > 0 \). As a result, \( \sigma_1^B(1|1, \gamma^B) = 1 \), contradicting \( \sigma_1^B(1|1, \gamma^B) \neq 1 \). Therefore, \( \sigma_1^B(1|1, \gamma^B) = 1 \).

**Claim 2** When the unbiased analyst keeps the default precision, i.e., \( \gamma^U = \bar{\gamma} \), any informative communication in the first period has to satisfy the following properties:

1. The investor updates favorably his belief about the analyst’s type when receiving report 0. More specifically,

\[
\Lambda(0, 1|\gamma^U = \bar{\gamma}, \gamma^B) \geq \Lambda(1, 1|\gamma^U = \bar{\gamma}, \gamma^B) \quad \text{and} \quad \Lambda(0, 0|\gamma^U = \bar{\gamma}, \gamma^B) \geq \Lambda(1, 0|\gamma^U = \bar{\gamma}, \gamma^B),
\]

and at least one of the inequalities is strict;

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(2) The unbiased analyst reports 0 when she observes signal 0, i.e. \( \sigma^U_1(0|\gamma^U = \bar{\gamma}, \gamma^B) = 0 \).

If she observes signal 1, the unbiased analyst reports 1 with positive probability, i.e. \( \sigma^U_1(1|\gamma^U = \bar{\gamma}, \gamma^B) > 0 \);

(3) The biased analyst reports 1 with positive probability for both signals. In addition, she reports 1 more often when she observes signal 1 than when she observes signal 0, i.e. \( \sigma^B_1(1|\gamma^U = \bar{\gamma}, \gamma^B) \geq \sigma^B_1(0|\gamma^U = \bar{\gamma}, \gamma^B) > 0 \).

I prove the result in nine steps. Each step identifies a property that must hold for first period informative communication when \( \gamma^U = \bar{\gamma} \leq \gamma^B \).

**Property 1-4.** The first four steps are the same as the proof of Claim 1.

**Property 5.** \( \Lambda(1, 1|\bar{\gamma}, \gamma^B) \leq \Lambda(0, 1|\bar{\gamma}, \gamma^B) \).

Prove by contradiction. Suppose instead \( \Lambda(1, 1|\bar{\gamma}, \gamma^B) > \Lambda(0, 1|\bar{\gamma}, \gamma^B) \). By Property 1, \( \Lambda(0, 0|\bar{\gamma}, \gamma^B) \geq \Lambda(1, 0|\bar{\gamma}, \gamma^B) \). Thus, \( \Pi^U_R(1|\bar{\gamma}, \gamma^B) < \Pi^U_R(0|\bar{\gamma}, \gamma^B) \) for \( J = B, U \). On the other hand, \( \Pi^L_C(1|\bar{\gamma}, \gamma^B) \geq \Pi^L_C(0|\bar{\gamma}, \gamma^B) \) for \( J = B, U \). Therefore, for \( J = B, U \), either \( \sigma^I_1(0|\bar{\gamma}, \gamma^B) = 0 \), or \( \sigma^I_1(1|\bar{\gamma}, \gamma^B) = 1 \). At the same time,

\[
\Lambda(1, 1|\bar{\gamma}, \gamma^B) > \Lambda(0, 1|\bar{\gamma}, \gamma^B) \\
\implies \phi^U_1(1|1) > \phi^B_1(1|1) \\
\implies \bar{\gamma}\sigma^U_1(1|\bar{\gamma}, \gamma^B) + (1 - \bar{\gamma})\sigma^U_1(0|\bar{\gamma}, \gamma^B) > \gamma^B\sigma^B_1(1|\bar{\gamma}, \gamma^B) + (1 - \gamma^B)\sigma^B_1(0|\bar{\gamma}, \gamma^B).
\]

\[ (9) \]

\[
\Lambda(0, 0|\bar{\gamma}, \gamma^B) \geq \Lambda(1, 0|\bar{\gamma}, \gamma^B) \\
\implies \phi^U_1(1|0) \leq \phi^B_1(1|0) \\
\implies \bar{\gamma}\sigma^U_1(0|\bar{\gamma}, \gamma^B) + (1 - \bar{\gamma})\sigma^U_1(1|\bar{\gamma}, \gamma^B) \leq \gamma^B\sigma^B_1(0|\bar{\gamma}, \gamma^B) + (1 - \gamma^B)\sigma^B_1(1|\bar{\gamma}, \gamma^B).
\]

\[ (10) \]

There are four subcases:

(a) \( \sigma^U_1(0|\bar{\gamma}, \gamma^B) = \sigma^B_1(0|\bar{\gamma}, \gamma^B) = 0 \).

In this case, (9) can be reduced to \( \bar{\gamma}\sigma^U_1(1|\bar{\gamma}, \gamma^B) > \gamma^B\sigma^B_1(1|\bar{\gamma}, \gamma^B) \), which implies that \( \sigma^U_1(1|\bar{\gamma}, \gamma^B) > \sigma^B_1(1|\bar{\gamma}, \gamma^B) \) because \( \gamma^B \geq \bar{\gamma} \). On the other hand, by (10), \( (1 - \bar{\gamma})\sigma^U_1(1|\bar{\gamma}, \gamma^B) \leq (1 - \gamma^B)\sigma^B_1(1|\bar{\gamma}, \gamma^B) \), which leads to \( \sigma^U_1(1|\bar{\gamma}, \gamma^B) \leq \sigma^B_1(1|\bar{\gamma}, \gamma^B) \). A contradiction.

(b) \( \sigma^U_1(1|\bar{\gamma}, \gamma^B) = \sigma^B_1(1|\bar{\gamma}, \gamma^B) = 1 \).
In this case, by (10),
\[ \bar{\gamma} \sigma_1^U(0|\bar{\gamma},\gamma^B) + 1 - \bar{\gamma} \leq \gamma^B \sigma_1^B(0|\bar{\gamma},\gamma^B) + 1 - \gamma^B \Rightarrow \bar{\gamma}(1 - \sigma_1^U(0|\bar{\gamma},\gamma^B)) \geq \gamma^B(1 - \sigma_1^B(0|\bar{\gamma},\gamma^B)) \]
\[ \Rightarrow \sigma_1^U(0|\bar{\gamma},\gamma^B) \leq \sigma_1^B(0|\bar{\gamma},\gamma^B). \]

On the other hand, (9) can be reduced to:
\[ \bar{\gamma} + (1 - \bar{\gamma})\sigma_1^U(0|\bar{\gamma},\gamma^B) > \gamma^B + (1 - \gamma^B)\sigma_1^B(0|\bar{\gamma},\gamma^B) \]
\[ \Rightarrow (\gamma^B - \bar{\gamma})(\sigma_1^U(0|\bar{\gamma},\gamma^B) - 1) + (1 - \gamma^B)[\sigma_1^U(0|\bar{\gamma},\gamma^B) - \sigma_1^B(0|\bar{\gamma},\gamma^B)] > 0 \]
\[ \Rightarrow \sigma_1^U(0|\bar{\gamma},\gamma^B) > \sigma_1^B(0|\bar{\gamma},\gamma^B). \]

Again a contradiction.

(c) \( \sigma_1^U(0|\bar{\gamma},\gamma^B) = 0 \) and \( \sigma_1^B(1|\bar{\gamma},\gamma^B) = 1 \).

In this case, (9) can be reduced to \( \bar{\gamma}\sigma_1^U(1|\bar{\gamma},\gamma^B) > \gamma^B + (1 - \gamma^B)\sigma_1^B(0|\bar{\gamma},\gamma^B) \), which leads to \( \sigma_1^U(1|\bar{\gamma},\gamma^B) > \frac{\gamma^B}{\bar{\gamma}} \geq 1 \). A contradiction.

(d) \( \sigma_1^U(1|\bar{\gamma},\gamma^B) = 1 \) and \( \sigma_1^B(0|\bar{\gamma},\gamma^B) = 0 \).

By (10),
\[ \bar{\gamma}\sigma_1^U(0|\bar{\gamma},\gamma^B) + 1 - \bar{\gamma} \leq (1 - \gamma^B)\sigma_1^B(1|\bar{\gamma},\gamma^B) \Rightarrow \begin{cases} \bar{\gamma}\sigma_1^U(0|\bar{\gamma},1) + 1 - \bar{\gamma} \leq 0, & \text{if } \gamma^B = 1 \\ \sigma_1^B(1|\bar{\gamma},\gamma^B) = 1 \text{ and } \sigma_1^U(0|\bar{\gamma},\gamma^B) = 0. & \text{if } \gamma^B = \bar{\gamma} \end{cases} \]

If \( \gamma^B = 1 \), \( \bar{\gamma}\sigma_1^U(0|\bar{\gamma},1) + 1 - \bar{\gamma} \leq 0 \). A contradiction. If \( \gamma^B = \bar{\gamma} \), then \( \sigma_1^B(1|\bar{\gamma},\bar{\gamma}) = 1 \) and \( \sigma_1^U(0|\bar{\gamma},\bar{\gamma}) = 0 \). Recall that we start by supposing \( \sigma_1^U(1|\bar{\gamma},\gamma^B) = 1 \) and \( \sigma_1^B(0|\bar{\gamma},\gamma^B) = 0 \) hold. Hence, (9) can be reduced to \( \bar{\gamma} > \gamma^B \). Another contradiction.

To summarize, \( \Lambda(1,1|\bar{\gamma},\gamma^B) > \Lambda(0,1|\bar{\gamma},\gamma^B) \) leads to a contradiction in every possible scenario. Hence, \( \Lambda(1,1|\bar{\gamma},\gamma^B) \leq \Lambda(0,1|\bar{\gamma},\gamma^B) \).

**Property 6.** \( \Lambda(0,0|\bar{\gamma},\gamma^B) \geq \Lambda(1,0|\bar{\gamma},\gamma^B) \) and \( \Lambda(1,1|\bar{\gamma},\gamma^B) \leq \Lambda(0,1|\bar{\gamma},\gamma^B) \), and at least one of these inequalities is strict.

By Property 1 and Property 5, \( \Lambda(0,0|\bar{\gamma},\gamma^B) \geq \Lambda(1,0|\bar{\gamma},\gamma^B) \) and \( \Lambda(1,1|\bar{\gamma},\gamma^B) \leq \Lambda(0,1|\bar{\gamma},\gamma^B) \). Suppose both hold with equality. Then, there is no reputational consequence of reporting 1, and \( \Gamma_1(1,\gamma) > \Gamma_1(0,\gamma) \) implies \( \sigma_1^B(1|\bar{\gamma},\gamma^B) = \sigma_1^B(0|\bar{\gamma},\gamma^B) = 1 \). Therefore, \( \Lambda(0,0|\bar{\gamma},\gamma^B) > \Lambda(1,0|\bar{\gamma},\gamma^B) \), contradicting \( \Lambda(0,0|\bar{\gamma},\gamma^B) = \Lambda(1,0|\bar{\gamma},\gamma^B) \).

**Property 7.** \( \sigma_1^U(0|\bar{\gamma},\gamma^B) = 0 \).

By Property 6, \( \Pi_1^U(0|\bar{\gamma},\gamma^B) > 0 \); by Property 2, \( \Pi_1^U(0|\bar{\gamma},\gamma^B) < 0 \). Hence, \( \sigma_1^U(0|\bar{\gamma},\gamma^B) = 0 \).
Property 8. $\sigma^U(1|\vec{\gamma}, \gamma^B) > 0$ and $\sigma^B(0|\vec{\gamma}, \gamma^B) \leq \sigma^B(1|\vec{\gamma}, \gamma^B)$.

These directly follow from Property 3, Property 4, and Property 7.

Property 9. $\sigma^B(0|\vec{\gamma}, \gamma^B) > 0$.

Prove by contradiction. Suppose that $\sigma^B(0|\vec{\gamma}, \gamma^B) = 0$. Then, $\Lambda(0, 0|\vec{\gamma}, \gamma^B) \geq \Lambda(1, 0|\vec{\gamma}, \gamma^B)$ (Property 1) and $\sigma^U(0|\vec{\gamma}, \gamma^B) = 0$ (Property 7) imply that:

$$
\Lambda(0, 0|\vec{\gamma}, \gamma^B) \geq \Lambda(1, 0|\vec{\gamma}, \gamma^B)
\Leftrightarrow \phi^U_1(1|0) \leq \phi^B_1(1|0)
\Leftrightarrow \tilde{\gamma} \sigma^U_1(0|\vec{\gamma}, \gamma^B) + (1 - \tilde{\gamma}) \sigma^U_1(1|\vec{\gamma}, \gamma^B) \leq \gamma^B \sigma^B_1(0|\vec{\gamma}, \gamma^B) + (1 - \gamma^B) \sigma^B_1(1|\vec{\gamma}, \gamma^B)
\Rightarrow (1 - \tilde{\gamma}) \sigma^U_1(1|\vec{\gamma}, \gamma^B) \leq (1 - \gamma^B) \sigma^B_1(1|\vec{\gamma}, \gamma^B). \tag{11}
$$

When $\gamma^B = 1$, by (11), $(1 - \tilde{\gamma}) \sigma^U_1(1|\gamma^B, 1) \leq 0$, which implies $\sigma^U_1(1|\gamma^B, 1) = 0$, contradicting $\sigma^U_1(1|\gamma^B, \gamma^B) > 0$ (Property 8).

When $\gamma^B = \tilde{\gamma}$, by (11), $\sigma^U_1(1|\gamma^B, \tilde{\gamma}) \leq \sigma^B_1(1|\gamma^B, \tilde{\gamma})$. On the other hand,

$$
\sigma^B_1(0|\vec{\gamma}, \tilde{\gamma}) = 0 \Rightarrow \phi^B_1(1|1) = \tilde{\gamma} \sigma^B_1(1|\gamma^B, \tilde{\gamma}) \text{ and } \phi^B_1(1|0) = (1 - \tilde{\gamma}) \sigma^B_1(1|\gamma^B, \tilde{\gamma}),
$$

and

$$
\sigma^U_1(0|\vec{\gamma}, \tilde{\gamma}) = 0 \Rightarrow \phi^U_1(1|1) = \tilde{\gamma} \sigma^U_1(1|\gamma^B, \tilde{\gamma}) \text{ and } \phi^U_1(1|0) = (1 - \tilde{\gamma}) \sigma^U_1(1|\gamma^B, \tilde{\gamma}).
$$

Hence,

$$
\frac{\phi^B_1(1|1)}{\phi^U_1(1|1)} = \frac{\sigma^B_1(1|\gamma^B, \tilde{\gamma})}{\sigma^U_1(1|\gamma^B, \tilde{\gamma})} = \frac{\phi^B_1(1|0)}{\phi^U_1(1|0)},
$$

and

$$
\frac{\phi^B_1(0|1)}{\phi^B_1(0|1)} = \frac{1 - \tilde{\gamma} \sigma^B_1(1|\gamma^B, \tilde{\gamma})}{1 - \tilde{\gamma} \sigma^U_1(1|\gamma^B, \tilde{\gamma})} \leq \frac{1 - (1 - \tilde{\gamma}) \sigma^B_1(1|\gamma^B, \tilde{\gamma})}{1 - (1 - \tilde{\gamma}) \sigma^U_1(1|\gamma^B, \tilde{\gamma})} = \frac{\phi^B_1(0|0)}{\phi^U_1(0|0)}.
$$

The inequality comes from $\sigma^U_1(1|\gamma^B, \tilde{\gamma}) \leq \sigma^B_1(1|\gamma^B, \tilde{\gamma})$. Therefore, $\Lambda(1, 1|\gamma^B, \tilde{\gamma}) = \Lambda(1, 0|\gamma^B, \tilde{\gamma})$ and $\Lambda(0, 1|\gamma^B, \tilde{\gamma}) \geq \Lambda(0, 0|\gamma^B, \tilde{\gamma})$. Then by the definition of $\Pi^B_B(\cdot)$, $\Pi^B_B(1|\gamma^B, \tilde{\gamma}) \geq \Pi^B_B(0|\gamma^B, \tilde{\gamma})$.

As a result, $\sigma^B_1(1|\gamma^B, \tilde{\gamma}) \leq \sigma^B_1(0|\gamma^B, \tilde{\gamma})$ (because $\Pi^B_B(1|\gamma^B, \tilde{\gamma}) = \Pi^B_B(0|\gamma^B, \tilde{\gamma})$). By $\sigma^B_1(0|\gamma^B, \gamma^B) = 0$, it must be true that $\sigma^B_1(1|\gamma^B, \tilde{\gamma}) = 0$. Then, $\Lambda(1, 1|\gamma^B, \tilde{\gamma}) = 1 > \Lambda(0, 1|\gamma^B, \tilde{\gamma})$, contradicting Property 5, $\Lambda(1, 1|\gamma^B, \gamma^B) \leq \Lambda(0, 1|\gamma^B, \gamma^B)$.

Therefore, for any $\gamma^B$, $\sigma^B(0|\gamma^B, \gamma^B) > 0$ has to hold in the first period informative communication.

Part I: If $\gamma^U = \gamma^B = 1$ and $x < \frac{(4 - 3\lambda)\Lambda}{4(2 - \lambda)^2}$, first period communication is babbling.
If $\gamma^U = \gamma^B = 1$, then any first period informative communication has to satisfy the properties characterized in Claim 1. Furthermore, $\sigma^B_1(0|1, 1) > 0$ has to hold. The reason is that: suppose not and instead $\sigma^B_1(0|1, 1) = 0$, then $\Lambda(0, 0|1, 1) = \Lambda(0, 1|1, 1) = \Lambda(1, 0|1, 1) = \Lambda(1, 0, 1, 1) = \lambda$, which contradicts property (1) in Claim 1.

Suppose first period informative communication obtains. Then, based on the analyst’s equilibrium communication strategy, the type inference function $\Lambda(m_1, w_1|1, 1)$ and the state inference function $\Gamma_1(m_1, 1, 1)$ are calculated according to (2) and (1). Thus,

$$
\Lambda(0, 0|1, 1) = \frac{\Lambda(1, 1|1, 1) = \lambda; \quad \Lambda(0, 1|1, 1) = 0;}{(1 - \sigma^B_1(0|1, 1))(1 - \lambda) + \lambda}, \quad \Lambda(1, 0|1, 1) = 0;
$$

$$
a_1(1, 1, 1) = \Gamma_1(1, 1, 1) = \frac{1}{1 + (1 - \lambda)\sigma^B_1(0|1, 1)}; \quad a_1(0, 1, 1) = \Gamma_1(0, 1, 1) = 0. \quad (12)
$$

Then:

$$
\Pi^B_C(0|1, 1) = -x\{a_1(1, 1, 1) - 1\}^2 - (a_1(0, 1, 1) - 1)^2 = x - x\left(\frac{1}{1 + (1 - \lambda)\sigma^B_1(0|1, 1)} - 1\right)^2,
$$

and

$$
\Pi^B_R(0|1, 1) = V^B(\Lambda(0, 0|1, 1), 1) - V^B(\Lambda(1, 0|1, 1), 1) = \frac{\lambda(4 - 3\lambda - 4(1 - \lambda)\sigma^B_1(0|1, 1))}{4(\lambda - 2 + 2(1 - \lambda)\sigma^B_1(0|1, 1))^2}.
$$

Treating $\sigma^B_1(0|1, 1)$ as a variable $\mu$, tedious algebra shows that

$$
\frac{\partial[\Pi^B_R(0|1, 1) - \Pi^B_C(0|1, 1)]}{\partial \mu}\bigg|_{\mu = \sigma^B_1(0|1, 1)} = 2(1 - \lambda)^2\left\{\frac{x\sigma^B_1(0|1, 1)}{[1 + (1 - \lambda)\sigma^B_1(0|1, 1)]^3} + \lambda(1 - \sigma^B_1(0|1, 1))}{[1 + (1 - 2\sigma^B_1(0|1, 1))(1 - \lambda)]^3}\right\} \geq 0.
$$

Hence,

$$
\Pi^B_R(0|1, 1) - \Pi^B_C(0|1, 1) \geq [(\Pi^B_R(0|1, 1) - \Pi^B_C(0|1, 1))|\sigma^B_1(0|1, 1) = 0] = -x + \frac{(4 - 3\lambda)\lambda}{4(2 - \lambda)^2}.
$$

Clearly, when $x < \frac{(4 - 3\lambda)\lambda}{4(2 - \lambda)^2}$, $\Pi^B_R(0|1, 1) - \Pi^B_C(0|1, 1) > 0$. Therefore, $\sigma^B_1(0|1, 1) = 0$. However, as argued above, if $\gamma^U = \gamma^B = 1$, $\sigma^B_1(0, 1) > 0$ has to hold in any informative communication. Therefore, if $x < \frac{(4 - 3\lambda)\lambda}{4(2 - \lambda)^2}$ and $\gamma^U = \gamma^B = 1$, communication in the first period must be uninformative.

**Part II:** If $\gamma^B \geq \gamma^U = \bar{\gamma}$ and $x \to 0$, first period communication is babbling.

---

27 The out-of-equilibrium beliefs play a crucial role here. Recall that I adopt the convention that the investor assigns prior probability to the analyst being unbiased when the posterior beliefs are undefined.
According to property (1) in Claim 2, if communication were informative, then both types of analysts would have reputational incentives to report 0. When $x \to 0$, analysts’ current incentives diminish. Therefore, when $x \to 0$, both analysts will report 0, leading to an uninformative communication.

**Proof of Proposition 1**

According to Claim 1, if $\gamma^U = 1 > \gamma^B = \tilde{\gamma}$, then in any informative communication, it must be true that: (1) $\sigma_I(0|1, \tilde{\gamma}) = 0$ and $\sigma_I(1|1, \tilde{\gamma}) = 1$; and (2) $\sigma_I^B(1|1, \tilde{\gamma}) = 1$. Claim 1 is silent, however, regarding $\sigma_I^B(0|1, \tilde{\gamma})$. To save on notation, I define $\sigma_I^B(0|1, \tilde{\gamma}) \equiv \mu \in [0, 1]$. Note that $\mu$ is a function of $x$ (I suppress $x$ when there is no scope for confusion).

Based on the communication strategy, the type inference function $\Lambda(m_1, w_1|1, \tilde{\gamma})$ and the state inference function $\Gamma_1(m_1, 1, \tilde{\gamma})$ are calculated according to (2) and (1). Thus

$$
\Lambda(1, 1|1, \tilde{\gamma}) = \frac{\lambda}{(\tilde{\gamma}(1-\mu) + \mu)(1-\lambda) + \lambda}; \quad \Lambda(0, 1|1, \tilde{\gamma}) = 0;
$$

$$
\Lambda(0, 0|1, \tilde{\gamma}) = \frac{\lambda}{\gamma(1-\mu)(1-\lambda) + \lambda}; \quad \Lambda(1, 0|1, \tilde{\gamma}) = 0;
$$

$$
a_1(1, 1, \tilde{\gamma}) = \Gamma_1(1, 1, \tilde{\gamma}) = \frac{\gamma + (1 - \tilde{\gamma})\mu(1-\lambda) + \lambda}{(1 + \mu)(1-\lambda) + \lambda};
$$

$$
a_1(0, 1, \tilde{\gamma}) = \Gamma_1(0, 1, \tilde{\gamma}) = \frac{(1 - \tilde{\gamma})(1-\mu)(1-\lambda)}{(1-\mu)(1-\lambda) + \lambda}.
$$

(13)

Then the biased analyst’s current reporting incentive and reputational reporting incentive when she observes signal 0 are calculated, respectively, using (13):

$$
\Pi_C^B(0|1, \tilde{\gamma}) = -x \{ (a_1(1, 1, \tilde{\gamma}) - 1)^2 - (a_1(0, 1, \tilde{\gamma}) - 1)^2 \},
$$

and

$$
\Pi_R^B(0|1, \tilde{\gamma}) = \tilde{\gamma}[V^B(\Lambda(0, 0|1, \tilde{\gamma}), 1) - V^B(\Lambda(1, 0|1, \tilde{\gamma}), 1)] + (1 - \tilde{\gamma})[V^B(\Lambda(0, 1|1, \tilde{\gamma}), 1) - V^B(\Lambda(1, 1|1, \tilde{\gamma}), 1)].
$$

Both $\Pi_R^B(0|1, \tilde{\gamma})$ and $\Pi_C^B(0|1, \tilde{\gamma})$ are continuous in $\mu$. Clearly, if $x < \frac{\Pi_R^B(0|1, \tilde{\gamma})}{(a_1(0, 1, \tilde{\gamma}) - 1)^2 - (a_1(1, 1, \tilde{\gamma}) - 1)^2}$, then $\Pi_C^B(0|1, \tilde{\gamma}) < \Pi_R^B(0|1, \tilde{\gamma})$. As a result, $\mu = \sigma^B_I(0|1, \tilde{\gamma}) = 0$. Tedious algebra reduces the qualifier to $x < x = \frac{1}{\Pi_R^B(0|1, \tilde{\gamma})} \frac{(25 - 1)\lambda(\lambda + 4\gamma(1-\lambda))}{(a_1(0, 1, \tilde{\gamma}) - 1)^2 - (a_1(1, 1, \tilde{\gamma}) - 1)^2}$.

If $x > \frac{\Pi_R^B(0|1, \tilde{\gamma})}{(a_1(0, 1, \tilde{\gamma}) - 1)^2 - (a_1(1, 1, \tilde{\gamma}) - 1)^2}$, then $\Pi_C^B(0|1, \tilde{\gamma}) > \Pi_R^B(0|1, \tilde{\gamma})$. As a result, $\mu = \sigma^B_I(0|1, \tilde{\gamma}) = 1$. Then the qualifier can be reduced to $x > \tilde{x} \equiv \frac{(4 - 3\lambda)\lambda + 2\gamma(\lambda^2 - 2)}{8\lambda - 12}$.

If $x \leq \tilde{x} \leq \tilde{x}$, by continuity, there always exist a mixed communication strategy for the biased analyst when she observes signal 0. That is, $\mu = \sigma^B_I(0|1, \tilde{\gamma}) \in [0, 1]$ is such that $\Pi_C^B(0|1, \tilde{\gamma}) = \Pi_R^B(0|1, \tilde{\gamma})$.
Next, I show that if \( \gamma^U = 1 > \gamma^B = \bar{\gamma} \), informative communication obtains in the first period for any \( x \). Suppose the investor holds conjectures that:

\[
\begin{align*}
\hat{\sigma}_1^U(0|1, \bar{\gamma}) &= 0, \quad \hat{\sigma}_1^U(1|1, \bar{\gamma}) = 1, \quad \hat{\sigma}_1^B(0|1, \bar{\gamma}) = \mu, \quad \hat{\sigma}_1^B(1|1, \bar{\gamma}) = 1,
\end{align*}
\]

where \( \mu \) depends on \( x \) in the following way: if \( x < \underline{x}, \mu = 0; \) if \( x > \bar{x}, \mu = 1; \) if \( \underline{x} \leq x \leq \bar{x}, \mu \in (0,1) \) is such that \( \Pi^B_U(0|1, \bar{\gamma}) = \Pi^B_R(0|1, \bar{\gamma}) \). Based on such beliefs, the type inference function and the state inference function are derived as in (13). Now I examine whether indeed the analysts’ best responses are consistent with the investor’s conjectures.

(i) When the unbiased analyst observes signal 1:

\[
\begin{align*}
\Pi^U_C(1|1, \bar{\gamma}) &= -x(a_1(1, 1, \bar{\gamma}) - 1)^2 + x(a_1(0, 1, \bar{\gamma}) - 1)^2 > 0, \\
\Pi^U_R(1|1, \bar{\gamma}) &= V^U(\Lambda(0, 1|1, \bar{\gamma}), 1) - V^U(\Lambda(1, 1|1, \bar{\gamma}), 1) < 0.
\end{align*}
\]

Therefore, \( \sigma^U_C(1|1, \bar{\gamma}) = 1 \), consistent with the investor’s conjecture.

(ii) When the unbiased analyst observes signal 0:

\[
\begin{align*}
\Pi^U_C(0|1, \bar{\gamma}) &= -x(a_1(1, 1, \bar{\gamma}) - 0)^2 + x(a_1(0, 1, \bar{\gamma}) - 0)^2 < 0, \\
\Pi^U_R(0|1, \bar{\gamma}) &= V^U(\Lambda(0, 0|1, \bar{\gamma}), 1) - V^U(\Lambda(1, 0|1, \bar{\gamma}), 1) > 0.
\end{align*}
\]

Therefore, \( \sigma^U_C(0|1, \bar{\gamma}) = 0 \), consistent with the investor’s conjecture.

(iii) When the biased analyst observes signal 0:

\[
\begin{align*}
\Pi^B_C(0|1, \bar{\gamma}) &= -x(a_1(1, 1, \bar{\gamma}) - 1)^2 + x(a_1(0, 1, \bar{\gamma}) - 1)^2 > 0, \\
\Pi^B_R(0|1, \bar{\gamma}) &= \bar{\gamma}[V^B(\Lambda(0, 0|1, \bar{\gamma}), 1) - V^B(\Lambda(0, 0|1, \bar{\gamma}), 1)] + (1 - \bar{\gamma})[V^B(\Lambda(1, 0|1, \bar{\gamma}), 1) - V^B(\Lambda(1, 1|1, \bar{\gamma}), 1)] \\
&= \bar{\gamma}[V^B(\Lambda(0, 0|1, \bar{\gamma}), 1) - V^B(\Lambda(1, 1|1, \bar{\gamma}), 1)] + (2\bar{\gamma} - 1)[V^B(\Lambda(1, 1|1, \bar{\gamma}), 1) - V^B(\lambda_2 = 0, 1)] > 0.
\end{align*}
\]

If \( x < \underline{x} \), then \( \mu = 0 \). Straightforward algebra shows that \( \Pi^B_C(0|1, \bar{\gamma}) < \Pi^B_R(0|1, \bar{\gamma}) \). Hence, \( \sigma^B_C(0|1, \bar{\gamma}) = 0 \), consistent with the investor’s conjecture.

If \( x > \bar{x} \), then \( \mu = 1 \). Simplifying terms shows that \( \Pi^B_C(0|1, \bar{\gamma}) > \Pi^B_R(0|1, \bar{\gamma}) \). Therefore, \( \sigma^B_C(0|1, \bar{\gamma}) = 1 \), consistent with the investor’s conjecture.

If \( \underline{x} \leq x \leq \bar{x}, \mu \) is such that \( \Pi^B_C(0|1, \bar{\gamma}) = \Pi^B_R(0|1, \bar{\gamma}) \). Therefore, the biased analyst will randomize between announcing 0 and 1, and \( \sigma^B_C(0|1, \bar{\gamma}) = \mu \), consistent with the investor’s conjecture.
(iv) When the biased analyst observes signal 1:

\[
\Pi_B^U(1|\gamma) = \Pi_C^B(0|1, \gamma) > 0, \\
\Pi_B^R(1|\gamma) = \gamma[V^B(\Lambda(0, 1|1, \gamma), 1) - V^B(\Lambda(0, 1|1, \gamma), 1)] + (1 - \gamma)[V^B(\Lambda(0, 0|1, \gamma), 1) - V^B(\Lambda(1, 0|1, \gamma), 1)] < (1 - \gamma)[V^B(\Lambda(0, 1|1, \gamma), 1) - V^B(\Lambda(1, 1|1, \gamma), 1)] + \gamma[V^B(\Lambda(0, 0|1, \gamma), 1) - V^B(\Lambda(1, 0|1, \gamma), 1)] = \Pi_B^R(0|1, \gamma).
\]

If \(x < x\), then \(\mu = 0\). Then, \(\Lambda(0, 0|1, \gamma) = \Lambda(1, 1|1, \gamma) > \Lambda(1, 0|1, \gamma) = \Lambda(0, 1|1, \gamma)\). Hence,

\[
\Pi_B^R(1|\gamma) = \gamma[V^B(\Lambda(0, 1|1, \gamma), 1) - V^B(\Lambda(1, 1|1, \gamma), 1)] + (1 - \gamma)[V^B(\Lambda(0, 0|1, \gamma), 1) - V^B(\Lambda(1, 0|1, \gamma), 1)] = (2\gamma - 1)[V^B(\Lambda(0, 1|1, \gamma), 1) - V^B(\Lambda(1, 1|1, \gamma), 1)] < 0.
\]

Therefore, \(\Pi_B^R(1|\gamma) < 0 < \Pi_C^B(1|\gamma)\). As a result, \(\sigma_t^B(1|1, \gamma) = 1\), consistent with the investor’s conjecture.

If \(x \geq x\), then \(\mu \leq 1\), which implies that \(\Pi_C^B(0|1, \gamma) \geq \Pi_B^R(0|1, \gamma)\). Therefore,

\[
\Pi_B^R(1|\gamma) < \Pi_B^R(0|1, \gamma) \leq \Pi_C^B(0|1, \gamma) = \Pi_C^B(1|1, \gamma).
\]

Hence the biased analyst will report 1, again consistent with the investor’s conjecture.

To summarize, if \(\gamma^U = 1 > \gamma^B = \bar{\gamma}\), informative communication in the first period obtains for any \(x\).

\[\blacksquare\]

**Proof of Proposition 2**

As Section 3.1 and Proposition 1 have shown, if the investor conjectures that only the unbiased analyst becomes perfectly informed and communication in each period is informative, then the best responses of the analysts who have conjectured precision will indeed constitute informative communication. Now it remains to show that given the investor’s conjecture, for \(\frac{1 - \tilde{\epsilon}}{4} \equiv \Delta \leq c \leq \tilde{\Delta} \equiv \min\{\frac{1 - \tilde{\epsilon}}{2}, \frac{1}{8}\}\), it is indeed the unbiased analyst’s best response to become perfectly informed, while the biased analyst’s optimal choice is to keep the default precision.

(i) The unbiased analyst’s choice of precision:
To that end, first I examine the unbiased analyst’s precision effect in the second period. Note that $a_2(m_2, \lambda_2, \tilde{\gamma}^U)$ and $V^U(\lambda_2, \gamma^U, \tilde{\gamma}^U)$ are independent of $\tilde{\gamma}^B$ because the biased analyst always reports 1 in the second period.

If the investor conjectures that $\tilde{\gamma}^U = 1$, then the unbiased analyst with the conjectured precision $\gamma^U = 1$ will tell the truth. For the unbiased analyst with precision $\bar{\gamma}$, when she observes signal 0, she will compare her payoff conditional on sending message 0, $-\bar{\gamma}(a_2(0, \lambda_2, 1) - 1)^2 - (1 - \bar{\gamma})(a_2(0, \lambda_2, 1) - 0)^2$, with her payoff conditional on sending message 1, $-\bar{\gamma}(a_2(1, \lambda_2, 1) - 1)^2 - (1 - \bar{\gamma})(a_2(1, \lambda_2, 1) - 0)^2$. The difference is $[a_2(1, \lambda_2, 1) - a_2(0, \lambda_2, 1)][a_2(1, \lambda_2, 1) + a_2(0, \lambda_2, 1) - 2\bar{\gamma}] = a_2(1, \lambda_2, 1)(a_2(1, \lambda_2, 1) - 2\bar{\gamma}) < 0$. Hence the unbiased analyst with precision $\bar{\gamma}$ will report 1 when her signal is 1. On the other hand, if the unbiased analyst has precision $\bar{\gamma}$ and observes signal 0, her optimal reporting strategy may vary with parameter values, and hence I consider all possible scenarios below. If the unbiased analyst with off-equilibrium precision $\bar{\gamma}$ reports 0 when she observes signal 0, then

$$V^U(\lambda_2, \gamma^U = 1, \tilde{\gamma}^U = 1) - V^U(\lambda_2, \gamma^U = \bar{\gamma}, \tilde{\gamma}^U = 1)$$

$$= -\frac{1}{2}(1 - \bar{\gamma})[(a_2(1, \lambda_2, 1) - 1)^2 - (a_2(0, \lambda_2, 1) - 1)^2 + (a_2(0, \lambda_2, 1))^2 - (a_2(1, \lambda_2, 1))^2]$$

$$= (1 - \bar{\gamma})[a_2(1, \lambda_2, 1) - a_2(0, \lambda_2, 1)]$$

$$= \frac{1 - \bar{\gamma}}{2 - \lambda_2}.$$

If the unbiased analyst with off-equilibrium precision $\bar{\gamma}$ reports 1 when she observes signal 0, then

$$V^U(\lambda_2, \gamma^U = 1, \tilde{\gamma}^U = 1) - V^U(\lambda_2, \gamma^U = \bar{\gamma}, \tilde{\gamma}^U = 1)$$

$$= -\frac{1}{2}(a_2(0, \lambda_2, 1))^2 - (a_2(1, \lambda_2, 1))^2]$$

$$= \frac{1}{2(2 - \lambda_2)^2}.$$

Overall,

$$V^U(\lambda_2, \gamma^U = 1, \tilde{\gamma}^U = 1) - V^U(\lambda_2, \gamma^U = \bar{\gamma}, \tilde{\gamma}^U = 1)$$

$$\geq \min\left\{\frac{1 - \bar{\gamma}}{2 - \lambda_2}, \frac{1}{2(2 - \lambda_2)^2}\right\}$$

$$\geq \min\left\{\frac{1 - \bar{\gamma}}{2}, \frac{1}{8}\right\} \equiv \Delta. \quad (14)$$

Given the subsequent communication continuation game played in each period, the unbiased analyst’s utility at the information acquisition stage if she becomes better informed
is as follows (the arguments in \(a_1(\cdot)\) are \(m_1, \gamma^U, \text{and } \gamma^B\):

\[
U_0^U(\gamma^U = 1|\gamma^B = 1, \gamma^B = \gamma) = \frac{1}{2}[-x(a_1(1, 1, \gamma) - 1)^2 + V^U(\Lambda(1, 1|1, \gamma), 1, 1)] + \frac{1}{2}[-x(a_1(0, 1, \gamma))^2 + V^U(\Lambda(0, 0|1, \gamma), 1, 1)] - c.
\]

If the unbiased analyst keeps her default precision, it’s straightforward that

\[
U_0^U(\gamma^U = \gamma|\gamma^U = 1, \gamma^B = \gamma) < \frac{1}{2}[-x(a_1(1, 1, \gamma) - 1)^2 - x(a_1(0, 1, \gamma))^2] + \frac{1}{2} \max\{V^U(\Lambda(1, 1|1, \gamma), \gamma, 1), V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)\} + \frac{1}{2} \max\{V^U(\Lambda(0, 0|1, \gamma), \gamma, 1), V^U(\Lambda(1, 0|1, \gamma), \gamma, 1)\}.
\]

The reason is simply that \(\frac{1}{2}[-x(a_1(1, 1, \gamma) - 1)^2 - x(a_1(0, 1, \gamma))^2]\) is the maximal first period payoff for any first period communication strategy, and

\[
\frac{1}{2} \max\{V^U(\Lambda(1, 1|1, \gamma), \gamma, 1), V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)\} + \frac{1}{2} \max\{V^U(\Lambda(0, 0|1, \gamma), \gamma, 1), V^U(\Lambda(1, 0|1, \gamma), \gamma, 1)\}
\]

is the maximal second period expected utility for any first period communication strategy.

Define \(\Delta^U \equiv U_0^U(\gamma^U = 1|\gamma^U = 1, \gamma^B = \gamma) + c - U_0^U(\gamma^U = \gamma|\gamma^U = 1, \gamma^B = \gamma)\), then

\[
\Delta^U > \frac{1}{2}[V^U(\Lambda(1, 1|1, \gamma), 1, 1) - \max\{V^U(\Lambda(1, 1|1, \gamma), \gamma, 1), V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)\}] + \frac{1}{2}[V^U(\Lambda(0, 0|1, \gamma), 1, 1) - \max\{V^U(\Lambda(0, 0|1, \gamma), \gamma, 1), V^U(\Lambda(1, 0|1, \gamma), \gamma, 1)\}].
\]

If \(V^U(\Lambda(1, 1|1, \gamma), \gamma, 1) \geq V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)\),

\[
A = \frac{1}{2}[V^U(\Lambda(1, 1|1, \gamma), 1, 1) - V^U(\Lambda(1, 1|1, \gamma), \gamma, 1)] \geq \frac{1}{2} \min\{\frac{1 - \gamma}{2}, \frac{1}{8}\}.
\]

The inequality stems from (14).

On the contrary, if \(V^U(\Lambda(1, 1|1, \gamma), \gamma, 1) < V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)\),

\[
A = \frac{1}{2}[V^U(\Lambda(1, 1|1, \gamma), 1, 1) - V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)]
\]

\[
= \frac{1}{2}[V^U(\Lambda(1, 1|1, \gamma), 1, 1) - V^U(\Lambda(0, 1|1, \gamma), 1, 1)]
\]

\[
+ \frac{1}{2}[V^U(\Lambda(0, 1|1, \gamma), 1, 1) - V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)]
\]

\[
> \frac{1}{2}[V^U(\Lambda(0, 1|1, \gamma), 1, 1) - V^U(\Lambda(0, 1|1, \gamma), \gamma, 1)]
\]

\[
\geq \frac{1}{2} \min\{\frac{1 - \gamma}{2}, \frac{1}{8}\}.
\]
The first inequality stems from the fact that \( \frac{\partial V}{\partial \lambda_2} > 0 \) (same proof as the one showing that \( \frac{\partial V}{\partial \lambda_2(\lambda_2, \gamma)} > 0 \)) and \( \Lambda(1, 1|1, \tilde{\gamma}) > \Lambda(0, 1|1, \tilde{\gamma}) \); and the second inequality stems from (14).

Hence in all cases,

\[
A \geq \frac{1}{2} \min\{\frac{1 - \tilde{\gamma}}{2}, \frac{1}{8}\}.
\]

Analogously,

\[
B \geq \frac{1}{2} \min\{\frac{1 - \tilde{\gamma}}{2}, \frac{1}{8}\}.
\]

Therefore,

\[
\Delta^U > A + B \geq \min\{\frac{1 - \tilde{\gamma}}{2}, \frac{1}{8}\} \equiv \bar{\Delta}.
\]

When \( c \leq \bar{\Delta}, U^U_1(\gamma^U = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \gamma) - U^U_1(\gamma^U = \gamma|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \gamma) = \Delta^U - c > 0 \). Hence, the unbiased analyst will choose to acquire information.

(ii) The biased analyst’s choice of precision:

First, I introduce the analyst’s current reporting incentive and reputational reporting incentive when the analyst’s precision is unobservable. Write \( \Pi^U_C(s_1, \gamma^J|\tilde{\gamma}) \) for the current expected gain to the type J analyst of reporting 1, rather than reporting 0, given signal \( s_1 \), precision \( \gamma^J \), and the investor’s conjecture of analysts’ precision \( \tilde{\gamma} = (\tilde{\gamma}^U, \tilde{\gamma}^B) \). Therefore,

\[
\Pi^U_C(s_1, \gamma^U|\tilde{\gamma}) = -x\{\gamma^U(a_1(1, \tilde{\gamma}) - s_1)^2 + (1 - \gamma^U)(a_1(1, \tilde{\gamma}) - (1 - s_1))^2\}
+ x\{\gamma^U(a_1(0, \tilde{\gamma}) - s_1)^2 + (1 - \gamma^U)(a_1(0, \tilde{\gamma}) - (1 - s_1))^2\},
\]

\[
\Pi^B_C(1, \gamma^B|\tilde{\gamma}) = \Pi^B_C(0, \gamma^B|\tilde{\gamma}) = -x(a_1(1, \tilde{\gamma}) - 1)^2 + x(a_1(0, \tilde{\gamma}) - 1)^2.
\]

Write \( \Pi^U_R(s_1, \gamma^J|\tilde{\gamma}) \) for the expected reputational gain to the type J analyst of reporting 0 rather than reporting 1, when she observes signal \( s_1 \) and has precision \( \gamma^J \) and the investor’s conjecture of analysts’ precision is \( \tilde{\gamma} \). Hence,

\[
\Pi^U_R(s_1, \gamma^U|\tilde{\gamma}) = \gamma^U[V^U(\Lambda(0, s_1|\tilde{\gamma}), \gamma^U, \tilde{\gamma}) - V^U(\Lambda(1, s_1|\tilde{\gamma}, \gamma^U, \tilde{\gamma})]
+ (1 - \gamma^U)[V^U(\Lambda(0, 1 - s_1|\tilde{\gamma}), \gamma^U, \tilde{\gamma}) - V^U(\Lambda(1, 1 - s_1|\tilde{\gamma}, \gamma^U, \tilde{\gamma})],
\]

\[
\Pi^B_R(s_1, \gamma^B|\tilde{\gamma}) = \gamma^B[V^B(\Lambda(0, s_1|\tilde{\gamma}), \tilde{\gamma}) - V^B(\Lambda(1, s_1|\tilde{\gamma}, \tilde{\gamma})]
+ (1 - \gamma^B)[V^B(\Lambda(0, 1 - s_1|\tilde{\gamma}), \tilde{\gamma}) - V^B(\Lambda(1, 1 - s_1|\tilde{\gamma}, \tilde{\gamma})].
\]

Thus a type J analyst has a strict incentive to report 1 when she observes \( s_1 \) and has precision \( \gamma^J \) if and only if \( \Pi^U_C(s_1, \gamma^J|\tilde{\gamma}) > \Pi^U_R(s_1, \gamma^J|\tilde{\gamma}) \).
Note that the biased analyst would have more reputational incentive to tell the truth when the precision is higher (because $\Lambda(0,0|1,\tilde{\gamma}) > \Lambda(1,0|1,\tilde{\gamma})$ and $\Lambda(0,1|1,\tilde{\gamma}) < \Lambda(1,1|1,\tilde{\gamma})$), while her current incentive to report 1 is independent of her precision. Formally,

$$\Pi_B^B(s_1 = 0, \gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) > \Pi_B^B(s_1 = 0, \gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma});$$

$$\Pi_B(s_1 = 1, \gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) < \Pi_B^B(s_1 = 1, \gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma});$$

$$\Pi_C^B(s_1, \gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) = \Pi_C^B(s_1, \gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma})$$

for any $s_1$.

As a result,

$$\sigma_1^B(s_1 = 0, \gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) \geq \sigma_1^B(s_1 = 0, \gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}),$$

$$\sigma_1^B(s_1 = 1, \gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) \leq \sigma_1^B(s_1 = 1, \gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}).$$

That is, having a higher precision would make the biased analyst more likely to tell the truth. According to similar argument as in Proposition 1, $\sigma_1^B(1, \tilde{\gamma}|1, \tilde{\gamma}) = 1$, hence $\sigma_1^B(1, 1|1, \tilde{\gamma}) = 1$. Therefore the following three cases are mutually exclusive and commonly exhaustive:

(I) If $x$ is sufficiently large such that $\sigma_1^B(0, 1|1, \tilde{\gamma}) = \sigma_1^B(0, \tilde{\gamma}|1, \tilde{\gamma}) = 1$. In this case, $\Delta_1^B = 0$ simply because the biased analyst’s report is independent of her signal.

(II) If $x < \bar{x}$, then by similar argument as in Proposition 1, $\sigma_1^B(0, \tilde{\gamma}|1, \tilde{\gamma}) = 0$. Therefore, a fortiori, $\sigma_1^B(0, 1|1, \tilde{\gamma}) = 0$. Recall that $\sigma_1^B(1, \tilde{\gamma}|1, \tilde{\gamma}) = \sigma_1^B(1, 1|1, \tilde{\gamma}) = 1$. That is, the biased analyst reports truthfully for each precision. Then the biased analyst’s utility at the information acquisition stage conditional on keeping the default precision is (here I add the previously suppressed argument $x$ in the $a_1(\cdot)$ and $\Lambda(\cdot)$ functions):

$$U_0^B(\gamma^B = \tilde{\gamma}|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) = \frac{1}{2}[-x(a_1(1, 1, \tilde{\gamma}, x) - 1)^2] + \frac{1}{2}[-x(a_1(0, 1, \tilde{\gamma}, x) - 1)^2] + \frac{1}{2}\tilde{\gamma}V^B(\Lambda(1, 1|1, \tilde{\gamma}, x), 1) + \frac{1}{2}(1 - \tilde{\gamma})V^B(\Lambda(0, 1|1, \tilde{\gamma}, x), 1) + \frac{1}{2}\tilde{\gamma}V^B(\Lambda(0, 0|1, \tilde{\gamma}, x), 1) + \frac{1}{2}(1 - \tilde{\gamma})V^B(\Lambda(1, 0|1, \tilde{\gamma}, x), 1).$$

The biased analyst’s utility at the information acquisition stage conditional on acquiring information is:

$$U_0^B(\gamma^B = 1|\tilde{\gamma}^U = 1, \tilde{\gamma}^B = \tilde{\gamma}) = \frac{1}{2}[-x(a_1(1, 1, \tilde{\gamma}, x) - 1)^2] + \frac{1}{2}[-x(a_1(0, 1, \tilde{\gamma}, x) - 1)^2] + \frac{1}{2}V^B(\Lambda(1, 1|1, \tilde{\gamma}, x), 1) + \frac{1}{2}V^B(\Lambda(0, 0|1, \tilde{\gamma}, x), 1) - c.$$
Then the benefit of becoming perfectly informed for the biased analyst is
\[
\Delta_B^B \equiv U_0^B(\gamma^B = 1|\overline{\gamma}^U = 1, \overline{\gamma}^B = \overline{\gamma}) - U_0^B(\gamma^B = \overline{\gamma}|\overline{\gamma}^U = 1, \overline{\gamma}^B = \overline{\gamma})
\]
\[
= \frac{1}{2}(1 - \overline{\gamma})\left[V_B(\Lambda(0, 0|1, \overline{\gamma}) - 1) - V_B(\Lambda(0, 1|1, \overline{\gamma}) - 1)\right]
\]
\[
+ \frac{1}{2}(1 - \overline{\gamma})\left[V_B(\Lambda(1, 0|1, \overline{\gamma}) - 1) - V_B(\Lambda(1, 1|1, \overline{\gamma}) - 1)\right].
\]

(III) For the intermediate values of \( x \), \( 1 \geq \sigma_B^B(0, \overline{\gamma}|1, \overline{\gamma}) > \sigma_B^B(0, 1|1, \overline{\gamma}) = 0 \). Then by reveal preference, the biased analyst’s utility when having precision \( \overline{\gamma} \) under the optimal communication strategy \( \sigma_B^B(0, \overline{\gamma}|1, \overline{\gamma}) \) is (weakly) greater than her utility under the truth-telling strategy. Hence the biased analyst’s benefit from acquiring information in this case is less than her benefit if the truth-telling strategy is implemented for both precision levels. Therefore,
\[
\Delta_{III}^B \leq \frac{1}{2}(1 - \overline{\gamma})[V_B(\Lambda(0, 0|1, \overline{\gamma}) - 1) - V_B(\Lambda(1, 0|1, \overline{\gamma}) - 1)]
\]
\[
+ \frac{1}{2}(1 - \overline{\gamma})[V_B(\Lambda(1, 1|1, \overline{\gamma}) - 1) - V_B(\Lambda(0, 1|1, \overline{\gamma}) - 1)].
\]

Therefore for any \( x \),
\[
\Delta^B \leq \frac{1}{2}(1 - \overline{\gamma})[V_B(\Lambda(0, 0|1, \overline{\gamma}) - 1) - V_B(\Lambda(1, 0|1, \overline{\gamma}) - 1)]
\]
\[
+ \frac{1}{2}(1 - \overline{\gamma})[V_B(\Lambda(1, 1|1, \overline{\gamma}) - 1) - V_B(\Lambda(0, 1|1, \overline{\gamma}) - 1)]
\]
\[
= \frac{1}{2}(1 - \overline{\gamma})\left[-\frac{(\Lambda(0, 0|1, \overline{\gamma}) - 1)^2}{(\Lambda(0, 0|1, \overline{\gamma}) - 1)^2} + \frac{1}{4} - \frac{(\Lambda(1, 1|1, \overline{\gamma}) - 1)^2}{(\Lambda(1, 1|1, \overline{\gamma}) - 1)^2} + \frac{1}{4}\right]
\]
\[
< \frac{1 - \overline{\gamma}}{4}.
\]

Hence if \( c \geq \Delta = \frac{1 - \overline{\gamma}}{4}, U_0^B(\gamma^B = 1|\overline{\gamma}^U = 1, \overline{\gamma}^B = \overline{\gamma}) - U_0^B(\gamma^B = \overline{\gamma}|\overline{\gamma}^U = 1, \overline{\gamma}^B = \overline{\gamma}) = \Delta^B - c < 0 \), the biased analyst will keep her default precision.

\[\square\]

Proof of Proposition 3

It’s straightforward that \( c^o \equiv \frac{1}{2}(1 - \overline{\gamma})\left[\frac{1}{2} - \frac{2\overline{\sigma}^2(1-\lambda)^2}{(\lambda + 2\overline{\gamma}(1-\lambda)^2)}\right] < \frac{1 - \overline{\gamma}}{4} = \Delta < \overline{\Delta}. \) The idea of the proof is to show that \( 1 \) If \( x < \min\{x^o, \frac{1}{2}\} \) and \( c < c^o \), then there exists an equilibrium in which \( \gamma^U = 1, \gamma^B = \overline{\gamma} \), first period communication is babbling and second period communication is informative. \( 2 \) Suppose \( \overline{\gamma} \geq 0.75 \). For \( x < \min\{x^o, \frac{1}{2}\} \) and \( c < c^o \), there is no other informative equilibria.
(1) Suppose the investor conjectures that $\tilde{\gamma}^U = 1$, $\tilde{\gamma}^B = \tilde{\gamma}$, first period communication is babbling, and second period communication is informative. Then the analysts’ best responses in each communication period are consistent with these conjectures. For the biased analyst, both her first period communication (babbling) and second period communication (always reports 1) are independent of her actual precision, hence she doesn’t have incentive to acquire information. However, the unbiased analyst’s benefit from acquiring information is $V^U(\lambda, 1, 1) > V^U(\lambda, \tilde{\gamma}, 1) > \Delta > c^o$, hence the unbiased analyst will acquire information. Therefore, for $x < \min\{x^o, \bar{x}\}$ and $c < c^o$, there exists an equilibrium in which $\gamma^U = 1, \gamma^B = \tilde{\gamma}$, first period communication is babbling and second period communication is informative.

(2) I prove this result by ruling out the impossible equilibria one by one. As shown in the main text, informative communication always obtains in the second period.

(i) Suppose in the postulated equilibria, the unbiased analyst keeps the default precision $\bar{\gamma}$. Then, in the second period, the unbiased analyst with the conjectured precision $\gamma$ will always tell the truth (similar arguments as in Section 3.1). In addition, the unbiased analyst with $\gamma^U = 1$ will also tell the truth. Hence,

$$V^U(\lambda_2, \gamma^U = 1, \tilde{\gamma}^U = \tilde{\gamma}) - V^U(\lambda_2, \gamma^U = \gamma, \tilde{\gamma}^U = \tilde{\gamma}) = -\frac{1}{2}(1 - \tilde{\gamma})[2a_2(1, \lambda_2, \tilde{\gamma}) - 1] = \frac{(1 - \gamma)(2\gamma - 1)}{2 - \lambda_2} \geq \frac{1 - \tilde{\gamma}}{4}. \tag{15}$$

The last inequality holds for $\tilde{\gamma} \geq 0.75$.

If first period communication is babbling, then the unbiased analyst’s benefit from acquiring information is $V^U(\lambda, 1, \tilde{\gamma}) > V^U(\lambda, \gamma, \tilde{\gamma}) \geq \frac{1 - \tilde{\gamma}}{4} > c^o$, therefore the unbiased analyst will have incentive to acquire information, upsetting the postulated equilibria.

If first period communication is informative, then according to Claim 2, $\sigma^U_1(0, 1|\gamma, \tilde{\gamma}^B) = 0$. In addition, $\Lambda(0, 0|\gamma, \tilde{\gamma}^B) = \Lambda(0, 0|\gamma, \tilde{\gamma}^B)$ and $a_1(1, \gamma, \tilde{\gamma}^B) = a_1(0, \gamma, \tilde{\gamma}^B)$, therefore $\Pi^U_2(0, 1|\gamma, \tilde{\gamma}^B) = V^U(\Lambda(0, 0|\gamma, \tilde{\gamma}^B), 1, \gamma) - V^U(\Lambda(0, 0|\gamma, \tilde{\gamma}^B), 1, \gamma) \geq 0 \text{ and } \Pi^U_3(0, 1|\gamma, \tilde{\gamma}^B) = -x(a_1(1, \gamma, \tilde{\gamma}^B) - 0)^2 + x(a_1(0, \gamma, \tilde{\gamma}^B) - 0)^2 < 0$. Hence $\sigma^U_1(0, 1|\gamma, \tilde{\gamma}^B) = 0$. Then the unbiased analyst’s utilities at the information acquisition stage (in the postulated equilibria)
are:

\[
U_0^U(\gamma^U = 1|\tilde{\gamma}^U = \tilde{\gamma}, \tilde{\gamma}^B) = \frac{1}{2}\sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B)[-x(a_1(1, \tilde{\gamma}, \tilde{\gamma}^B) - 1)^2 + V^U(\Lambda(1,1|\tilde{\gamma}, \tilde{\gamma}^B), 1, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}(1 - \sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B))[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B) - 1)^2 + V^U(\Lambda(0,1|\tilde{\gamma}, \tilde{\gamma}^B), 1, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B) - 0)^2 + V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), 1, \tilde{\gamma})] - c,
\]

\[
U_0^U(\gamma^U = \tilde{\gamma}|\tilde{\gamma}^U = \tilde{\gamma}, \tilde{\gamma}^B) = \frac{1}{2}\tilde{\gamma}\sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B)[-x(a_1(1, \tilde{\gamma}, \tilde{\gamma}^B) - 1)^2 + V^U(\Lambda(1,1|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}\tilde{\gamma}(1 - \sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B))[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B) - 1)^2 + V^U(\Lambda(0,1|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}(1 - \tilde{\gamma})[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B) - 1)^2 + V^U(\Lambda(0,1|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}\tilde{\gamma}[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B))^2 + V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}(1 - \tilde{\gamma})\sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B)[-x(a_1(1, \tilde{\gamma}, \tilde{\gamma}^B))^2 + V^U(\Lambda(1,0|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})]
\]

\[
+ \frac{1}{2}(1 - \tilde{\gamma})(1 - \sigma_1(1,1|\tilde{\gamma}, \tilde{\gamma}^B))[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B))^2 + V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})],
\]

where I ≡ Ia + Ib, III ≡ IIIa+IIIb+IIIc and IV ≡ IVa+IVb+IVc. It’s straightforward that

\[
IV < \frac{1}{2}[-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B))^2 + V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})],
\]

because \(-x(a_1(0, \tilde{\gamma}, \tilde{\gamma}^B))^2 > -x(a_1(1, \tilde{\gamma}, \tilde{\gamma}^B))^2, V^U(\Lambda_2, \tilde{\gamma}, \tilde{\gamma})\) is increasing in \(\lambda_2\) and \(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B) \geq \Lambda(1,0|\tilde{\gamma}, \tilde{\gamma}^B)\). Hence, by (15),

\[
II - IV > \frac{1}{2}[V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), 1, \tilde{\gamma}) - V^U(\Lambda(0,0|\tilde{\gamma}, \tilde{\gamma}^B), \tilde{\gamma}, \tilde{\gamma})] \geq \frac{1 - \tilde{\gamma}}{8}.
\]
If $\sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B) = 1$, then it must be true that $\Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B) < \Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B)$, that is, $V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) < -x(a_1(1,\bar{\gamma},\bar{\gamma}^B)-1)^2 + x(a_1(0,\bar{\gamma},\bar{\gamma}^B)-1)^2$. Hence,

$$I - III = \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[-x(a_1(1,\bar{\gamma},\bar{\gamma}^B) - 1)^2 + x(a_1(0,\bar{\gamma},\bar{\gamma}^B) - 1)^2] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] > \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] > \frac{1 - \bar{\gamma}}{8}.$$

If $\sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B) = 0$, then it must be true that $\Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B) > \Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B)$, that is, $V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) > -x(a_1(1,\bar{\gamma},\bar{\gamma}^B)-1)^2 + x(a_1(0,\bar{\gamma},\bar{\gamma}^B)-1)^2$. Hence,

$$I - III = \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[-x(a_1(0,\bar{\gamma},\bar{\gamma}^B) - 1)^2 + x(a_1(1,\bar{\gamma},\bar{\gamma}^B) - 1)^2] + \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] > \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] > \frac{1 - \bar{\gamma}}{8}.$$

If $0 < \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B) < 1$, then $\Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B) = \Pi^U_R(1,1|\bar{\gamma},\bar{\gamma}^B)$, that is, $V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) = -x(a_1(1,\bar{\gamma},\bar{\gamma}^B) - 1)^2 + x(a_1(0,\bar{\gamma},\bar{\gamma}^B) - 1)^2$. Hence,

$$I - III = \frac{1}{2} \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B)[V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(1,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] + \frac{1}{2}(1 - \bar{\gamma} \sigma^U_1(1,1|\bar{\gamma},\bar{\gamma}^B))[V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),1,\bar{\gamma}) - V^U(\Lambda(0,1|\bar{\gamma},\bar{\gamma}^B),\bar{\gamma},\bar{\gamma})] > \frac{1 - \bar{\gamma}}{8}.$$

Therefore, if the postulated equilibria is such that the unbiased analyst keeps the default precision, and first period communication is informative, then

$$U^0_U(\gamma^U = 1|\bar{\gamma}^U = \bar{\gamma},\bar{\gamma}^B) - U^0 U(\gamma^U = \bar{\gamma}|\bar{\gamma}^U = \bar{\gamma},\bar{\gamma}^B) > \frac{1 - \bar{\gamma}}{4} - c > \frac{1 - \bar{\gamma}}{4} - c^o > 0.$$
Hence, the unbiased analyst will have incentive to acquire information, again upsetting the postulated equilibria where the unbiased analyst keeps the default precision.

(ii) Suppose in the postulated equilibria, both analysts acquire information. Then by Lemma 1, when \( x < \min\{x^o, x\} \leq x^o \), the first period communication must be babbling. As a result, the biased analyst has no incentive to increase her precision because her communication in each period is independent of her actual precision. Thus there do not exist equilibria in which both analysts acquire information.

(iii) Suppose in the postulated equilibria, only the unbiased analyst becomes perfectly informed and communication in each period is informative. Then by similar argument as in Proposition 1, when \( x < \min\{x^o, x\} \leq x^o \), in the first period, both analysts report their signals truthfully. Hence \( \Lambda(0,0|1, \tilde{\gamma}) = \Lambda(1,0|1, \tilde{\gamma}) = \lambda \) and \( \Lambda(1,0|1, \tilde{\gamma}) = 0 \). Then the biased analyst’s benefit from acquiring information (by the proof of Proposition 2) is

\[
\Delta_{II}^B = \frac{1}{2}(1 - \tilde{\gamma})\{V^B(\Lambda(0,0|1, \tilde{\gamma}, x), \tilde{\gamma}^U = 1) - V^B(\Lambda(1,0|1, \tilde{\gamma}, x), \tilde{\gamma}^U = 1)\}
+ \frac{1}{2}(1 - \tilde{\gamma})\{V^B(\Lambda(1,1|1, \tilde{\gamma}, x), \tilde{\gamma}^U = 1) - V^B(\Lambda(0,1|1, \tilde{\gamma}, x), \tilde{\gamma}^U = 1)\}
= \frac{1}{2}(1 - \tilde{\gamma})[\frac{1}{2} - \frac{2\tilde{\gamma}^2(1-\lambda)^2}{(\lambda + 2\tilde{\gamma}(1-\lambda))^2}]
\equiv c^o.
\]

Hence for \( c < c^o \), the biased analyst has incentive to increase her precision, which upsets the postulated equilibria. □

**Proof of Corollary 2**

(1) Social welfare of having \( c = c_2 > c^o \) for \( x < \min\{x^o, x\} \).

According to Corollary 1, if \( x < \min\{x^o, x\} \) and \( c = c_2 > c^o \), the equilibrium is such that:

\[
\gamma^U^* = 1, \quad \gamma^B^* = \tilde{\gamma}, \quad \sigma^U_1(1,1) = \sigma^B_1(1, \tilde{\gamma}) = 1, \quad \sigma^U_1(0,1) = \sigma^B_1(0, \tilde{\gamma}) = 0,
\]

\[
\sigma^U_2(1,1) = 1, \quad \sigma^U_2(0,1) = 0, \quad \sigma^B_2(1, \tilde{\gamma}) = \sigma^B_2(0, \tilde{\gamma}) = 1,
\]

\[
\Lambda(1,1) = \Lambda(0,0) = \frac{\lambda}{\lambda + (1-\lambda)\gamma}, \quad \Lambda(0,1) = \Lambda(1,0) = 0.
\]

(note that in this proof, I suppress the dependence of \( \sigma^I_t(\cdot), a_t(\cdot) \) and \( \Lambda(\cdot) \) on \( \tilde{\gamma} \). Such conjecture is borne out in equilibrium.)

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Given this equilibrium, social welfare is calculated. To that end, I first calculate the investor’s expected utility for the second period (with posterior analyst reputation \( \lambda_2 \)):

\[
V^I(\lambda_2) = -Pr(w_2 = 1, m_2 = 1)[a_2(1, \lambda_2, 1) - 1]^2 - Pr(w_2 = 1, m_2 = 0)[a_2(0, \lambda_2, 1) - 1]^2 \\
- Pr(w_2 = 0, m_2 = 1)[a_2(1, \lambda_2, 1) - 0]^2 - Pr(w_2 = 0, m_2 = 0)[a_2(0, \lambda_2, 1) - 0]^2 \\
= \frac{1}{2}[a_2(1, \lambda_2, 1) - 1]^2 - \frac{1}{2}(1 - \lambda_2)[a_2(1, \lambda_2, 1) - 0]^2 - \frac{1}{2}\lambda_2[a_2(0, \lambda_2, 1) - 0]^2 \\
= -\frac{1 - \lambda_2}{2(2 - \lambda_2)}.
\]

It’s easy to show that \( V^I(\lambda_2) \) is increasing and convex in \( \lambda_2 \).

Note that in this equilibrium, both types of analysts truthfully report their signals in the first period. This amounts to that the investor observes the signal himself in the first period with (average) precision \( \lambda + (1 - \lambda)\bar{\gamma} \). Hence \( a_1(1) = \lambda + (1 - \lambda)\bar{\gamma} \) and \( a_1(0) = 1 - a_1(1) = (1 - \lambda)(1 - \bar{\gamma}) \), and the investor’s ex ante expected utility is as follows:

\[
EU^I(c = c_2) = \frac{1}{2}[\lambda + (1 - \lambda)\bar{\gamma}][-x(a_1(1) - 1)^2 + V^I(\Lambda(1, 1))] \\
+ \frac{1}{2}(1 - \lambda)(1 - \bar{\gamma})[-x(a_1(0) - 1)^2 + V^I(\Lambda(0, 1))] \\
+ \frac{1}{2}(1 - \lambda)(1 - \bar{\gamma})[-x(a_1(1) - 0)^2 + V^I(\Lambda(1, 0))] \\
+ \frac{1}{2}[\lambda + (1 - \lambda)\bar{\gamma}][-x(a_1(0) - 0)^2 + V^I(\Lambda(0, 0))] \\
= -\frac{x}{2}\{[\lambda + (1 - \lambda)\bar{\gamma}][a_1(1) - 1)^2 + a_1(0)^2] + (1 - \lambda)(1 - \bar{\gamma})[(a_1(0) - 1)^2 + a_1(1)^2]\} \\
+ (1 - \lambda)(1 - \bar{\gamma})V^I(0) + (\lambda + (1 - \lambda)\bar{\gamma})V^I(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}) \\
= -xa_1(1)a_1(0) + (1 - \lambda)(1 - \bar{\gamma})V^I(0) + (\lambda + (1 - \lambda)\bar{\gamma})V^I(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}).
\]

The unbiased analyst’s expected utility is:

\[
EU^U(c = c_2) = \frac{1}{2}[-x(a_1(1) - 1)^2 + V^U(\Lambda(1, 1), 1, 1) - x(a_1(0) - 0)^2 + V^U(\Lambda(0, 0), 1, 1)] - c_2 \\
= -\frac{x}{2}[(a_1(1) - 1)^2 + a_1(0)^2] + V^U(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}, 1, 1) - c_2 \\
= -xa_1(0)^2 + V^U(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}, 1, 1) - c_2.
\]
The biased analyst’s expected utility is:

\[ EU^B(c = c_2) = \frac{1}{2} \gamma [-x(a_1(1) - 1)^2 + V^B(\Lambda(1, 1), 1) - x(a_1(0) - 1)^2 + V^B(\Lambda(0, 0), 1)] \]

\[ + \frac{1}{2}(1 - \bar{\gamma})[-x(a_1(1) - 1)^2 + V^B(\Lambda(1, 0), 1) - x(a_1(0) - 1)^2 + V^B(\Lambda(0, 1), 1)] \]

\[ = - \frac{x}{2} [(a_1(1) - 1)^2 + (a_1(0) - 1)^2] + \bar{\gamma}V^B\left(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}, 1\right) + (1 - \bar{\gamma})V^B(0, 1) \]

Adding the players’ utilities, social welfare of having \( c = c_2 \) equals:

\[ W(c = c_2) = EU^I(c = c_2) + \lambda EU^U(c = c_2) + (1 - \lambda)EU^B(c = c_2). \]

(2) Social welfare of having \( c = c_1 \) for \( x < \min\{x^o, \bar{x}\} \).

According to Proposition 3, if \( \bar{\gamma} \geq 0.75 \), \( x < \min\{x^o, \bar{x}\} \) and \( c = c_1 < c^o \), then the unique informative equilibrium is such that \( \gamma^U^* = 1 \) and \( \gamma^B^* = \bar{\gamma} \), and only the second communication is informative. Based on this equilibrium, the investor’s ex ante expected utility is calculated analogously:

\[ EU^I(c = c_1) = -\frac{x}{4} + V^I(\lambda). \]

The unbiased analyst’s expected utility is:

\[ EU^U(c = c_1) = -\frac{x}{4} + V^U(\lambda, 1, 1) - c_1. \]

The biased analyst’s expected utility is:

\[ EU^B(c = c_1) = -\frac{x}{4} + V^B(\lambda, 1). \]

Adding up all players’ utilities, social welfare of having \( c = c_1 \) is

\[ W(c = c_1) = EU^I(c = c_1) + \lambda EU^U(c = c_1) + (1 - \lambda)EU^B(c = c_1). \]

(3) Now I compare the the social welfare of having \( c = c_1 \) and \( c = c_2 \).

For the investor,

\[ EU^I(c = c_2) - EU^I(c = c_1) \]

\[ = \sqrt[4]{1 - a_1(1)a_1(0)} + (1 - \lambda)(1 - \bar{\gamma})V^I(0) + (\lambda + (1 - \lambda)\bar{\gamma})V^I\left(\frac{\lambda}{\lambda + (1 - \lambda)\bar{\gamma}}, 1\right) - V^I(\lambda). \]
The fact that the first period difference is greater than 0 comes directly from \( a_1(1) + a_1(0) = 1 \); and the second period difference being greater than 0 is due to the convexity of \( V^I \).

For the unbiased analyst,

\[
EU^U(c = c_2) - EU^U(c = c_1) = x \left( \frac{1}{4} - a_1(0)^2 \right) + V^U(\frac{\lambda}{\lambda + (1 - \lambda)\gamma}, 1, 1) - V^U(\lambda, 1, 1) - (c_2 - c_1),
\]

first period, \( > 0 \)

second period, \( > 0 \)

cost difference, \( \to 0 \)

For the biased analyst,

\[
EU^B(c = c_2) - EU^B(c = c_1) = -x \left( a_1(1)^2 + a_1(0)^2 - \frac{1}{2} \right) - \frac{(1 - \gamma)\lambda^2[(4 - 3\lambda)\lambda + 4\gamma(1 - \lambda)(3 - 2\lambda)]}{4(2 - \lambda)^2(2\gamma(1 - \lambda) + \lambda)^2},
\]

first period, \( < 0 \)

second period, \( < 0 \)

Social welfare:

\[
W(c = c_2) - W(c = c_1) = \frac{1}{2} x \lambda(a_1(1) - a_1(0)) - \lambda(c_2 - c_1).
\]

first period, \( > 0 \)

cost difference, \( \to 0 \)

Thus, social welfare increases when information-gathering costs increase from \( c_1 \) to \( c_2 \).

\[\blacksquare\]
9 References


Fang, L. and Yasuda, A. (2005), Analyst reputation, underwriting pressure, and forecast accuracy, working paper, University of Pennsylvania.


