ADAPTATION AND USE OF RELIABILITY CONCEPTS IN INTERNAL CONTROL EVALUATION

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ABSTRACT

Earlier analytical studies in Internal Control Evaluation modeling have concentrated on the use of reliability theory in aggregating component reliabilities into a system reliability measure. In this study, we provide justification for the use of reliability theory for such aggregation. Furthermore, a model is developed for estimating component reliabilities from evidence on factors which are identified here. Another model is developed which uses system reliability as input and develops estimates of error probability distributions for audit planning purposes. This demonstrates the use of system reliability in audit planning.

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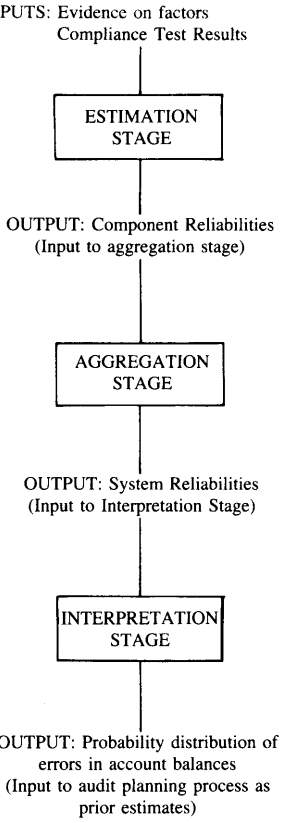
INTRODUCTION

The evaluation of internal control systems is an important aspect of auditing. AICPA's second standard of field work mandates internal control evaluation. SAS 20 [AICPA, 1980] requires communication of material internal control weaknesses to management. These standards and the passage of the Foreign Corrupt Practices Act of 1977 underscore the importance that the auditing profession gives to internal control evaluation.

The decision process involved in assessing the strengths and weaknesses of internal control systems has long been considered subjective. Consequently, advances in quantification and systematization of these processes have been taken up only recently by audit researchers. Research in this area has been of two types. Empirical research [Ashton, 1974, 1982; Joyce, 1976; Gaunmitz et al., 1982] has focussed on collecting evidence on how auditors currently make inter-

Figure 1. Stages of Decision Making in Internal Control Evaluation



2. Aggregation Stage:[2] In this stage, the component reliabilities estimated in the preceding stage are aggregated into a system reliability measure using the structure function.[3]
3. Interpretation Stage: In this stage, the system reliabilities computed in the preceding section are used to determine the net effect on the substantive testing plan of different accounts.

The earlier developments have concentrated on the application of reliability theory to the aggregation stage. There has been no attempt to systematically analyze the estimation and interpretation stages. In this study, an attempt is made to build a mathematical model to help the estimation process. Admittedly, such estimation cannot be modeled in a strict mathematical framework. The idea of building a mathematical model is not to replace the current subjective process employed but to gain additional insights to help make the estimation process more accurate. A more detailed discussion of this is given in the section on

implications. In this study, we also briefly discuss the application of reliability theory to the aggregation stage and provide some ideas of how the system reliability can be used in the interpretation stage.

Section 2 deals with the adaptation of reliability concepts to internal control evaluation. Section 3 deals with the estimation model and section 4 deals with the aggregation and interpretation stages. Section 5 presents the implications of the study and some concluding remarks.

ADAPTATION OF RELIABILITY CONCEPTS

Technical Overview of Reliability Theory

In this subsection, we deal with the definitions and results from reliability theory which are relevant. The discussion is for any general system, not necessarily for internal control system. The adaptation of these concepts to the internal control system is dealt with in the next subsection.

A system is a network of interrelated components. For each component, there needs to be a defined measure of success. The state of a component is determined by whether the component is "successful" or "not successful." The reliability of a component is defined as the probability of the component being found in a "success" state.[4] Similarly, the state of the whole system is also a binary variable with two possible values, "success" and "failure." The part of reliability theory which is of interest to us is the one which relates the system reliability to component reliabilities.

Consider a system with n components. The state of each component i is defined by a binary state variable x_i such that $x_i = 1$ if the component is successful and $x_i = 0$ if the component is not successful. Let the state of the system be represented by a binary variable B such that $B = 1$ if the system is successful and $B = 0$ if the system is not successful. For reliability theory to be applicable, it should be possible to represent B as an explicit function of the variables $x_1, x_2, \ldots, x_n$, i.e., $B = B(x_1, x_2, \ldots, x_n)$. This function is defined as

the documentation of the transaction. An error in the identification of the transaction is a validity error controlled for by population controls. An error in the documentation is an accuracy error controlled for by accuracy controls. For the entry in the source document to be correct, the following logic needs to be satisfied:

{The transaction is correctly identified as valid in the first place
or
the population control is successful}
AND
{The documentation is correct
or
the accuracy control is successful}

The identification and documentation phases of the activity can now be viewed as two independent components. Each population control pertaining to the identification phase can be represented as a component in parallel to the identification phase component of the activity. Similarly, each accuracy control can be represented as a component in parallel with the documentation phase component of the activity. The structure function of the above reliability system leads to the same logic as the error logic stated above. The representation as reliability network and the resulting structure function are shown in Figure 2. The probability of a control introducing an additional error is assumed to be zero.

We can now consider a transaction cycle such as the Purchase Cycle. The major activities in this cycle are: (1) Purchase Ordering (Purchase Order preparation), (2) Receipt of materials (Receiving Report preparation), and (3) Vouchering (Entry in Voucher Register and Voucher preparation). For the final entry (ledger posting) to be correct, each one of these major activities should be correct, i.e., these activity components have a series structure. As previously discussed, the population and accuracy controls are in parallel with the corresponding phases of the activities. The whole purchase transaction cycle can now

the same conclusion about the system. In this manner, the first assumption is justified.[5]

The second assumption seems to constrain the evaluation to only those population and accuracy controls which can be represented as components in a reliability framework. However, it is possible to work around this assumption if we treat those controls which cannot be so represented (such as, duty segregation, personnel competence, task complexity) as determinants of component reliabilities. It is in this context that the estimation stage becomes all the more important. If the use of reliability method is to be justified, it needs to be supported by an estimation stage in which the component reliabilities are estimated on the basis of (i) such personnel, task and environment related controls which cannot be represented as parallel controls, and (ii) the results of compliance testing.

The third assumption is about the usefulness of the system reliability measure. The objective of internal control evaluation as given in the standards is to provide a basis for deciding the audit sample size (extent), the time of substantive auditing for each account (timing) and the mix of analytical review and detailed testing (nature). These audit planning decisions require a prior estimate of the probability distribution of errors in each account. (For a discussion of audit planning, see Kinney and Warren [1979] and Felix and Grimlund [1976]). The interpretation stage of decision making mentioned earlier deals with the relationship between system reliabilities and the (prior) estimate of the probability distribution of errors. In this study, in section 4, some of those relations are modeled. This established the usefulness of reliability measure in auditing.

It must be noted that this justification for the use of reliability method in internal control evaluation is valid only if supported by appropriate estimation stage and interpretation stage models. As stated earlier, one of the primary objectives of this study is to provide these two linkages which make the use of reliability theory justifiable.

The Estimation Model

Estimation stage is that stage of decision making in which the evidence on environmental, personnel related and task related factors are used to estimate component reliabilities. We identify some of the factors which are relevant to such estimation here. These factors influence some parameters of the model. The manner in which the factors influence the parameters is presented in terms of a general logic because strict mathematical relationships do not exist. Then the determination of the component reliabilities from the parameters is presented.

In effect, we have tried to identify the purely subjective aspects of estimation in determining the parameters of the model. The relationships between the parameters and the component reliability estimates are developed in a quantitative manner. We stress again that the purpose of developing this model is not to

6. V_{1j}: This is the disutility (resulting from an organizational sanction or punishment) to the employee if he commits an error which is detected and is found to be intentionally committed. This parameter is an explicit representation of the punishment and reward system in the organization as it relates to errors.

7. V_{2j}: This is the disutility to the employee if he commits an error which is detected but is perceived to be unintentional. Usually, there is no formal punishment that is associated with such an error. However, the employee suffers a disutility because of a possible embarrassment and adverse reflection on his or her competence.

8. $g(U_j)$: This is a probability distribution which represents the author's perception of the integrity of the client organization employees. It gives the probability that at or above any given level of expected payoff U_j, the employee switches from being honest to committing an error. Again, it is not a strictly estimable distribution but helps to direct the attention of the auditor to making an assessment of the integrity of the employees.

9. P_{ij}: The probability of intentional error which we relate in the model to the other parameters listed above.

Table 1 gives the relationships between the factors and the above parameters. Most of these relationships can only be statements of directions but are still useful in aiding the estimation process.

Expected payoff to employee j, based on the above parameters, is

Table 1. Factor Parameter Relationships

	Parameters						
	P_{uj}	P_{1j}	P_2	B_j	V_{1j}	V_{2j}	$g(U_j)$
1. Segregation of duties	−1	−1	−1				
2. Degree of Centralization		−1	0		+1	+1	
3. Reward of Punishment system			0	0	0	0	
4. Personnel Competence	−1	−1	0	0			
5. Personnel Awareness	−1	−1	0	0			
6. Personnel Integrity							0
7. Task Complexity	+1	+1	+1				
8. Fatigue	+1	+1	+1				

The parameters as defined in the text are the following:

P_{uj} : Probability of unintentional error

P_{1j} : Probability of detection of an error (conditional)

P_2 : Conditional probability given that there is an error and it is detected, it would be seen as intentional

B_j : Expected payoff to the employee if an error is present (conditional)

V_{1j}, V_{2j} : Disutility to the employee if the detected error is found to be intentional or unintentional respectively

$g(U_j)$: Auditor's perception of employee integrity

NOTE: +1 INDICATES INCREASE; −1 INDICATES DECREASE; 0 INDICATES A POSITIVE OR A NEGATIVE INFLUENCE.

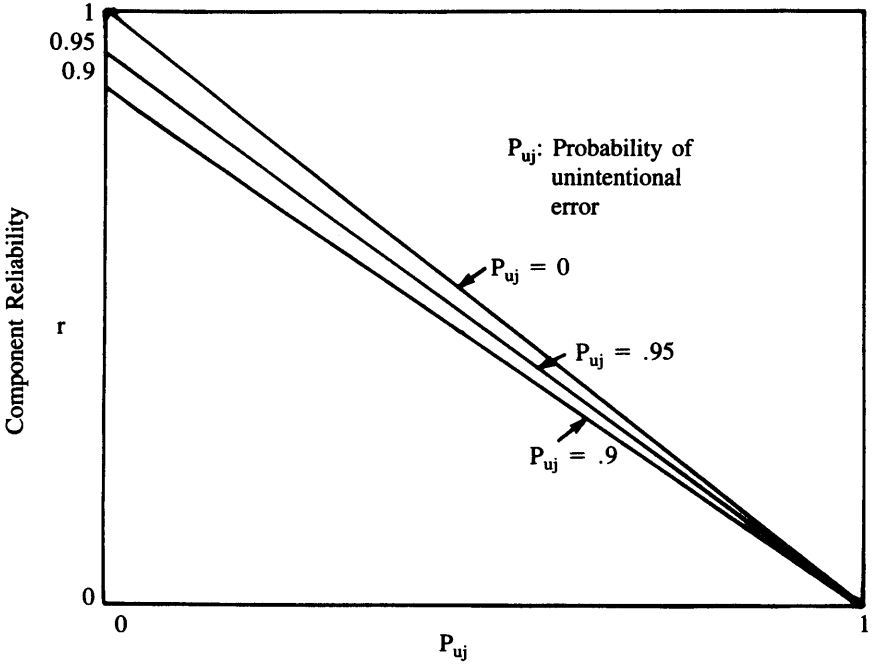
$$U_j = (1 - P_{1j}) B_j - P_{1j} (P_2 V_{1j} + P_3 V_{2j}) \quad (1)$$

Given this expression, we can now define a critical value P_{1jc} such that

$$P_{1jc} = B_j / [B_j + \{ V_{2j} + P_2 (V_{1j} - V_{2j}) \}] \quad (2)$$

With [illegible] which results in a conditional

Figure 4. Variation of Reliability r with Probabilities of Unintentional and Intentional Errors



system of checks and balances, the competence and awareness of employees etc. Component reliabilities can then be estimated using (4).

AGGREGATION AND INTERPRETATION MODELS

Aggregation Model

The aggregation of estimated component reliabilities into a system reliability measure has been studied in detail in the earlier studies. It has also been partially presented in section 2 while dealing with reliability theory. It is presented here briefly for the sake of completeness.

An activity is represented as two components, one for the identification phase and one for the documentation phase. The component reliability of the identification phase of the activity is the probability that the task is validly identified. The reliability of the documentation phase is the probability that the transaction has

been accurately recorded in the source document. In cases where separate estimation of population and accuracy controls for the same activity is deemed impractical, an approximation can be made by treating the activity as a single component with its reliability defined as the joint probability that it is validly identified and that it is accurately performed.

The reliability of a control is the joint probability that the control is applied (compliance) and that it is effective (design). This definition applies to both population and accuracy controls.

The controls are placed in parallel with the activities whose validity or accuracy they control. We can treat every activity with all its controls as a module. Each such module in the system is placed in series for the determination of system reliability.

Let there be n modules in the system. Denote modules by the index i. Denote the controls in module i as {i1, i2, . . . ik}. Let r_i represent component reliability of activity i and let r_{ij} represent the component reliability of the j^{th} control of activity i. Then, the reliability of module i is given by:

$$R_i = 1 - (1 - r_i)(1 - r_{i1})(1 - r_{i2}) \ldots . (1 - r_{ik}) \quad (5)$$

The system reliability R is given by:

$$R = R_1.R_2. \ldots . R_n \quad (6)$$

It is possible that a control applies to more than one activity. In such a case, that control appears in all the modules to which it applies.

Expressions (5) and (6) assume independence between all the components of the system. Usually, if the segregation of duties is adequate, this assumption is justified. For cases with inadequate segregation of duties, the aggregation procedure is more complicated and to some extent, subjective. Srinidhi [1984, 1988] deals with the problem of inadequate segregation of duties. There can be other types of dependencies but these can be taken care of within the aggregation

$$\text{mean} = M = E(e1 + e2) = E(e1) + E(e2) \tag{11}$$

$$\text{and variance } V = V(e1 + e2) = V(e1) + V(e2) \tag{12}$$

In general, if the account is affected by m transaction cycles, it will have a total error which is normally distributed with

$$\text{mean} = M = \sum_i N_i \cdot (1 - P_i) \cdot u_i \tag{13}$$

$$\text{and variance } V = \sum_i N_i \cdot (1 - P_i) \cdot [v_i + P_i \cdot u_i^2]. \tag{14}$$

If the materiality limits are judged to be $(-T_1, +T_2)$ for the error in the account, the probability of there being no material error in the account is given by

$$\int_{-T_1}^{+T_2} N(M, V)\, de \tag{15}$$

In deriving the above combination rule, normal distribution was assumed for the magnitude of errors and the reliability estimate was considered a point estimate. Similar combination rules can be derived if the reliability estimates are beta distributed and the magnitudes are normally distributed. Some of these are given in Srinidhi [1984].

IMPLICATIONS AND CONCLUDING REMARKS

In this study, we sought to provide a mathematical representation of the internal control evaluation process. In doing so, we built upon earlier studies which had divided the decision process into three separable but interconnected decision stages—estimation stage, aggregation stage and interpretation stage. We provided justification for using reliability theory to model the aggregation stage. This justification required the support of estimation and interpretation stages which had not been modeled earlier. In modeling the estimation stage, we identified some of the factors on which the auditors need to collect evidence, developed parameters to be estimated and mathematically related the parameters to component reliabilities. These component reliabilities were then used in the aggregation stage to estimate the system reliability for each transaction cycle. In the interpretation stage, we demonstrated the use of system reliability numbers in developing prior probability distribution of errors in account balances. These prior distributions are needed in planning the extent, nature and timing of substantive tests.

Though we have provided mathematical relationships wherever possible, these are meant only as guidelines in decision making and not intended to replace

subjectivity in decision making. For example, we have developed parameters in the estimation stage such as the perceived benefit of an error to the client employee and the distribution of an "integrity" variable. These cannot be estimated in a strict quantitative manner but the presence of these variables in the model should guide the auditor to think along these lines while making assessments of possible errors and irregularities. In contrast, the aggregation stage is mathematically much more adaptable to reliability theory and there is also empirical justification [Srinidhi and Vasarhelyi 1987] for directly using the mathematical model as a decision aid. The interpretation stage presented here involved some assumptions on distributions which can be empirically verified. The overall

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